UNIT-II[nd] :-> CURRENT ELECTRICITY

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Chapter -> 3

CURRENT ELECTRICITY

ELECTRIC CURRENT :->

When the charges flow in a conductor from one place to another in a particular direction, then this flow of charge is called 'electric current'.

'The rate of flow of charge through any section of the wire is known as electric current.'

It is a scalar quantity denoted by 'I'.

Thus,
$$I = \frac{\text{total charge flowing}}{\text{time taken}}$$

$$\boxed{I = \frac{q}{t}}$$ It is the steady current i.e., rate of flow of charge does not change with time.

If the current is not constant

$$\boxed{I = \frac{dq}{dt}}$$

=> The direction of the flow of current is opposite to the direction of flow of electrons.

SI Unit -> Cs^{-1} = Ampere (A)

$$\boxed{1 \text{ Ampere} = \frac{1 \text{ Coulomb}}{1 \text{ second}}}$$

1 Ampere :->

'The current flowing through a conductor is said to be 1 ampere if a charge of 1C passes through its given cross section in 1 second.'

NOTE :-> Although electric current has both magnitude and direction, yet it is a scalar quantity. This is because the laws of ordinary algebra are used to add electric currents and the laws of vector addition are not applicable to the addition of electric currents.

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⇒ Mechanism of Flow of Electric Current in a Conductor :→

In an atom of a substance, the electrons in the orbits close to the nucleus are bound to it under the strong attraction of the nucleus, but the electrons far from the nucleus experience a very feeble force. A few electrons leave their atoms and move freely within the substance (in the vacant space b/w the atoms). These electrons, called free electrons or conduction electrons, carry the electrical energy in the substance from one place to another. Therefore, the electrical conductivity of a solid substance depends upon the number of free electrons in it.

Drift Velocity and Relaxation Time

The free electrons in a conductor are always in **random motion** due to the thermal energy of the conductor. Likewise the velocity of free electrons due to the thermal energy of the conductor is called as thermal velocity.

The free electrons keep on suffering the collisions with the +ve ions in the conductor so frequently that the net flow of the electrons in any particular direction is zero. Hence, average thermal velocity of free electrons in a conductor is **zero**.

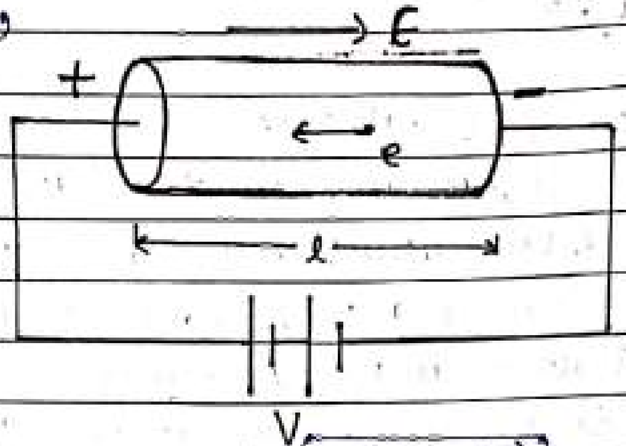
If $\vec{u_1}, \vec{u_2}, \ldots \vec{u_n}$ are random thermal velocities of n electrons in a conductor, then average thermal velocity of electrons i.e.,

$$\frac{\vec{u_1} + \vec{u_2} + \vec{u_3} + \cdots + \vec{u_n}}{n} = 0 \qquad \text{—— (1)}$$

Let a potential difference V is applied across the ends of the conductor of length l and E is the electric field set ~~up~~ up, then

$$\boxed{E = \frac{V}{l}} \qquad \text{—— (2)}$$

Then the force experienced by each free electron in the conductor,



$$\vec{F} = q\vec{E} \Rightarrow \boxed{\vec{F} = -e\vec{E}} \quad \text{--- (3)}$$

(-ve sign is there as electron experience a force in a direction opposite to the direction of applied electric field.)

If m is the mass of each electron then acceleration produced in each electron.

$$\boxed{\vec{a} = \frac{\vec{F}}{m} = \frac{-e\vec{E}}{m}} \quad \text{--- (4)}$$

Thus, the electrons get accelerated and acquire a velocity (opposite $\vec{E}$) in addition to their thermal velocities. The electrons accelerate and get deflected on suffering collision with +ve ions in the conductor. The electron, then starts afresh, with a random thermal velocity at each collision.

The short time, for which a free electron accelerates before it undergoes a collision with the +ve ion in the conductor is called Relaxation Time (τ).

Thus, the velocities acquired by electrons in the conductor will be

$$\vec{V_1} = \vec{u_1} + \vec{a}\tau_1 \ ; \ \vec{V_2} = \vec{u_2} + \vec{a}\tau_2 \ , \ \text{-------} \ \vec{V_n} = \vec{u_n} + \vec{a}\tau_n$$

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The average velocity with which the free electrons in a conductor get drifted under the influence of an external electric field applied across the ends of the conductor, is called as drift velocity ($\vec{V_d}$).

Thus,

$$\vec{V_d} = \frac{\vec{V_1} + \vec{V_2} + \vec{V_3} + \cdots + \vec{V_n}}{n}$$

$$= \frac{\vec{u_1} + \vec{a}\tau_1 + \vec{u_2} + \vec{a}\tau_2 + \cdots + \vec{u_n} + \vec{a}\tau_n}{n}$$

$$= \frac{\vec{u_1} + \vec{u_2} + \cdots + \vec{u_n}}{n} + \vec{a}\frac{(\tau_1 + \tau_2 + \cdots + \tau_n)}{n}$$

$$= 0 + \left(\frac{-e\vec{E}}{m}\right)\tau \quad [\text{from } (1) \ \& \ (4)]$$

Thus, drift velocity of free electrons is given by

$$\boxed{\vec{V_d} = \frac{-e\vec{E}}{m}\tau}$$

Here $\tau = \frac{\tau_1 + \tau_2 + \tau_3 + \cdots + \tau_n}{n}$ is called average relaxation time. (avg^n time taken by an e^- b/w 2 consecutive collision)
The -ve sign indicates that the electrons will drift in a direction opposite to the direction of applied electric field.

Relation b/w Electric Current and Drift Velocity

If A is the area of cross section, l is the length of the conductor then,

$$\text{volume of the conductor} = Al \quad \text{—— } (1)$$

If there are n electrons in the unit volume of the conductor then,

$$\text{Total no. of the electrons in the conductor} = nAl \quad \text{—— } (2)$$

As the charge on one electron is e, therefore total charge on all the free electrons in the conductor

$$q = nAle \quad \text{—— } (3)$$

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As $I = \frac{q}{t}$ $\therefore I = \frac{nAle}{t} = nAe\left(\frac{l}{t}\right)$

$$\boxed{I = neAv_d} \text{ — (4)} \quad (\text{as } l/t = v)$$

<u>NOTE:</u> → as $v_d = \frac{eE\tau}{m}$ (in magnitude)

∴ from eqⁿ (4) $\boxed{I = \frac{nAe^2E\tau}{m}}$ — (5)

<u>ELECTRON MOBILITY</u>

"It is the drift velocity acquired by electrons per unit strength of applied electric field across the conductor".

It is ~~dnot~~ denoted by (μ).

Thus,

$$\boxed{\mu = \frac{v_d}{E}} \text{ — (1)}$$

as $v_d = \frac{eE\tau}{m}$ — (2)

∴ from (1) & (2)

$$\boxed{\mu = \frac{e\tau}{m}} \text{ — (2)}$$

as $I = neAv_d$ & from (2) $v_d = \mu E$

$$\therefore \boxed{I = neA\mu E} \text{ — (3)}$$

<u>SI Unit</u> ⇒ $\frac{m/s}{V/m} = m^2V^{-1}s^{-1}$

<u>Dimensions</u> ⇒ $[\mu] = \frac{[v_d]}{[E]} = \frac{[LT^{-1}]}{[MLT^{-3}A^{-1}]}$

$$\boxed{[\mu] = [M^{-1}T^2A]}$$

CURRENT DENSITY

"The current density at a point in a conductor is the ratio of the current at that point in the conductor to the area of cross section of the conductor at that point."

It is denoted by (j).

$$\boxed{j = \frac{I}{A}} \quad \text{—— (1)}$$

as $I = neAv_d$

$$\therefore \quad \boxed{j = nev_d} \quad \text{—— (2)}$$

as $v_d = \frac{eE\tau}{m}$ $\quad \therefore \quad \boxed{j = \frac{ne^2E\tau}{m}} \quad \text{—— (3)}$

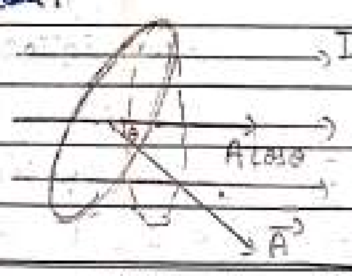
as $v_d = \mu E$, so from eqn (2)

$$\boxed{j = ne\mu E} \quad \text{—— (4)}$$

NOTE :-

(1) Current density (j) is the characteristic property of a point inside the conductor not of whole conductor.

$j = I/A\cos\theta \Rightarrow I = jA\cos\theta$



(2) Current density is a vector quantity.

In vector form :- $\boxed{i = \vec{j} \cdot \vec{A}}$

Its direction is same as that of current.

Q:- In a hydrogen atom an electron moves in an orbit of radius 0.5 Å with a speed of 2.2×10^6 m/s. Find the equivalent current.

Soln $r = 0.5\text{Å} = 0.5 \times 10^{-10}$ m $\qquad v = 2.2 \times 10^6$ m/s

$$I = \frac{q}{t} = \frac{q}{\frac{2\pi r}{v}} = \frac{qv}{2\pi r} = \frac{ev}{2\pi r}$$

$$I = \frac{(1.6 \times 10^{-19}) \times (2.2 \times 10^6)}{2 \times \frac{22}{7} \times 5 \times 10^{-11}} = \frac{7 \times 8 \times 10^{-20} \times 10^5 \times 10^{11}}{5}$$

$$I = 11.2 \times 10^{-4} = 1.12 \times 10^{-3} = 1.12\,\text{mA}$$

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Q:) An aluminium wire of diameter 0.24 cm is connected in series to a copper wire of diameter 0.16 cm. The wires carry the current of 10A. Find :-

(i) current density in Al wire

(ii) drift velocity in Cu wire

given :) no. of e^-s per unit volume in Cu wire is 8.4×10^{28}

(i) $j = \frac{I}{A} = \frac{I}{\pi r^2} = \frac{I}{\pi \frac{d^2}{4}} = \frac{4I}{\pi d^2}$

$= \frac{4 \times 10}{3.14 \times (24 \times 10^{-4})^2}$

$j = 2.2 \times 10^6 \; A/m^2$

(ii) $V_d = \frac{I}{neA} = \frac{10}{8.4 \times 10^{28} \times 1.6 \times 10^{-19} \times 3.14 \times (8 \times 10^{-4})^2}$

$V_d = 3.7 \times 10^{-4} \; m/s$

Q:) The current in a wire varies with time according to the relation $I = 3 + 2t$ where I is in Ampere and t is in seconds. How many coulomb of charge pass across the cross section of wire in time 0 to 4 sec. What constant current would transport the same charge in the same interval of the time.

Soln :) (a) as I is not constant

$\therefore I = \frac{dq}{dt}$

$dq = I\,dt$

$q = \int I\,dt$

$q = \int_0^4 (3+2t)\,dt$

$q = [3t]_0^4 + \left[\frac{2t^2}{2}\right]_0^4$

$q = 12 + 16 = 28\,C$

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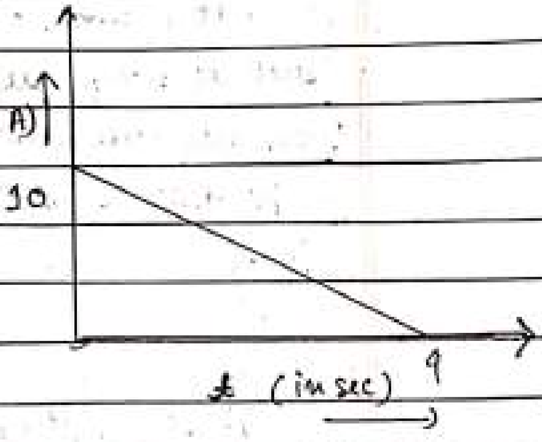
(b) $I = \frac{q}{t}$ (for constant current) $= \frac{28}{4} = 7A$

Q⇒ Find the total charge flowing through the wire from I-t graph shown below:→



Sol⇒ as $q = It$

$= $ area under I-t graph

$= \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 4 \times 10 = 20C$.

Q⇒ Estimate the average drift speed of conduction electrons in a Cu wire of cross sectional area $1 \times 10^{-7} m^2$, carrying a current of 1·5A. Assume that each copper atom contributes roughly one conduction electron. The density of copper is $9.0 \times 10^3 kg\, m^{-3}$ and its atomic mass is 63·5u. Take Avogadro's no. $= 6.0 \times 10^{23} mol^{-1}$.

Sol⇒ Mass of $1 m^3$ of Cu $= 9.0 \times 10^3 kg = 9 \times 10^6 g$

1 mol of Cu i.e. 63·5g of Cu contains $= 6.0 \times 10^{23}$ atoms.

So, $9 \times 10^6 g$ $(1m^3)$ of Cu contains $= \frac{6 \times 10^{23}}{63.5} \times 9 \times 10^6$ atoms

$= 8.5 \times 10^{28}$ atoms

Number of conduction electrons,

$n = $ number of Cu atoms $= 8.5 \times 10^{28}$

Now, $I = 1.5A$, $A = 10^{-7} m^2$, $e = 1.6 \times 10^{-19} C$

$\therefore V_d = \frac{I}{enA} = \frac{1.5}{(1.6 \times 10^{-19}) \times (8.5 \times 10^{28}) \times (10^{-7})}$

$V_d = \frac{15}{16 \times 85 \times 10} = 1.1 \times 10^{-3} ms^{-1}$

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OHM'S LAW

It states that - "if the physical conditions of the conductor (such as temperature, pressure, etc.) remain unchanged, then the current flowing through a conductor is directly proportional to the potential difference applied across its ends."

i.e.,
$$V \propto I$$
$$\boxed{V = RI} \quad — (1)$$

where, R is the constant of proportionality, called electrical resistance of the conductor.

⇒ It is independent of V and I.

from eqn (1)

Resistance of Conductor ⇒ $\boxed{R = \frac{V}{I}}$

Therefore, electrical resistance of a conductor is defined as:-
"The ratio of the potential difference applied across the ends of the conductor to the current flowing through it."

NOTE:- The conductors which obeys Ohm's law are called ohmic conductors.

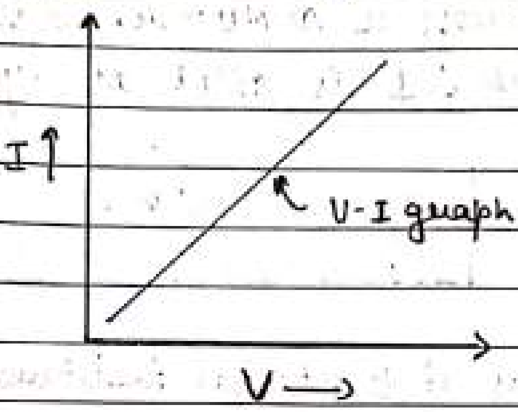
Unit of Resistance:- As $R = \frac{V}{I}$

∴ SI unit of Electrical Resistance is volt ampere^{-1} (VA^{-1})

$$1 \text{ volt ampere}^{-1} = 1 \text{ ohm } (\Omega) = \frac{1 \text{ volt}}{1 \text{ ampere}}$$

Thus, "the electrical resistance of a conductor is said to be one ohm if on applying a potential difference of 1 volt across its ends, the current flowing through it is one ampere."

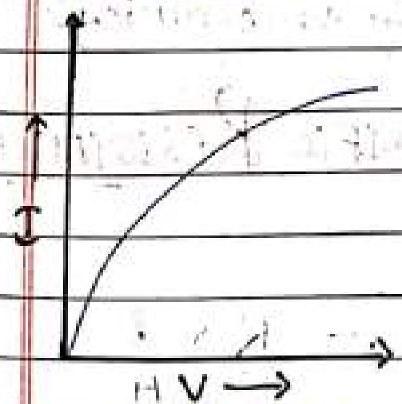
⇒ For ohmic conductors the V-I graph is a straight line.



Cause of Resistance :→ As discussed earlier when a potential difference is applied across the ends of a conductor, the free electrons in it get accelerated. As the electrons moves, they collide against the ions and the atoms and their motion is thus opposed. This opposition is termed as electrical resistance of the conductor.

Exception of Ohm's Law : Non-linear V-I Characteristics :→

Ohm's law is not a fundamental law of nature. In many cases the V-I graph is not a straight line. For liquid electrolytes, vaccum tubes, p-n junction diodes, thermistors, transistors and thermionic values, etc. Ohm's law does not hold. For these, the ratio V/I, that is R is not constant, but depends upon the applied potential difference. Such circuit components are called non-ohmic.



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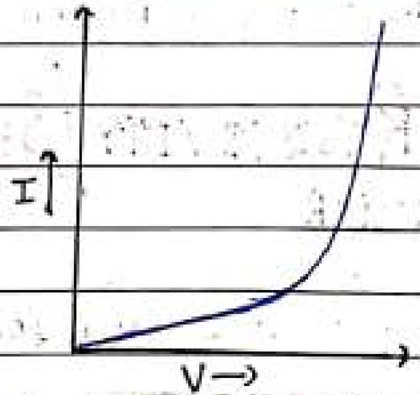
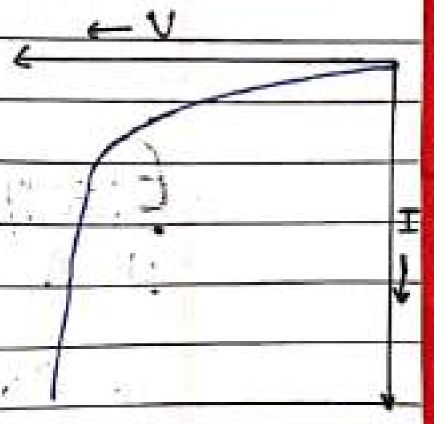
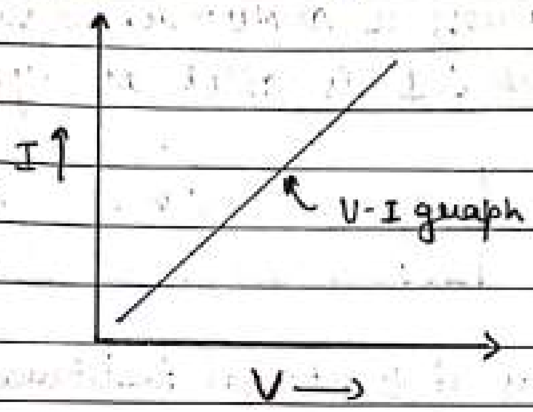


For p-n junction diode (Forward Biasing)



For a p-n junction diode (Backward Biasing).

=) For Ohmic Conductors the V-I graph is a straight line.



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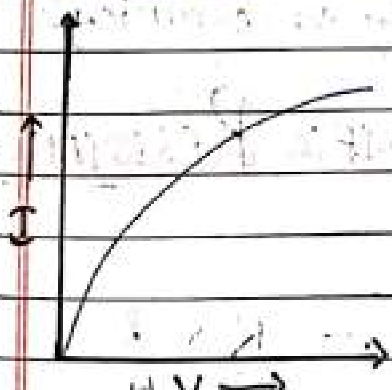

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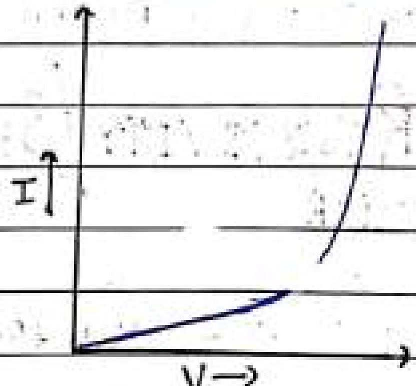

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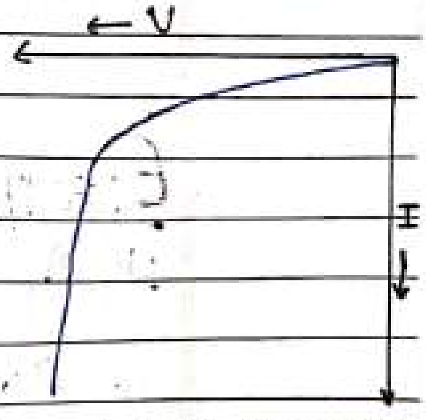

For a p-n junction diode (Backward Biasing).

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'The ratio of small change in potential difference ΔV, applied across a conductor, to the small change produced in current ΔI is called as dynamic resistance.'

Thus,

$$\boxed{\text{Dynamic Resistance} = \frac{\Delta V}{\Delta I}}$$

Dimensions of Electrical Resistance :→

$$[R] = \frac{[V]}{[I]} = \frac{[ML^2T^{-3}A^{-1}]}{[A]} = [ML^2T^{-3}A^{-2}]$$

⇒ FACTORS AFFECTING ELECTRICAL RESISTANCE OF A CONDUCTOR

The electrical resistance of a conductor is :→

(a) Directly proportional to the length of the conductor (l) i.e., $\boxed{R \propto l}$

(b) Inversely proportional to the area of cross section of the conductor (A) i.e., $\boxed{R \propto \frac{1}{A}}$

(c) Directly proportional to the temperature of the conductor (T) i.e., $\boxed{R \propto T}$

Besides these, the resistance of the conductor also depends upon the nature of the material of the conductor.

ELECTRICAL RESISTIVITY / SPECIFIC RESISTANCE OF A CONDUCTOR

As $R \propto l$ and $R \propto \frac{1}{A}$ therefore, $R \propto \frac{l}{A}$

$$\boxed{R = \rho \frac{l}{A}} \quad —(1)$$

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Here, 'ρ' is a constant, called electrical resistivity or specific resistance of the conductor. Its value depends upon the nature of the material and temperature of the conductor but not on the dimensions of the conductor.

$\therefore$ From eqn (1) $\boxed{\rho = R\frac{A}{l}}$ (2)

If $l = 1$ and $A = 1$, then from eqn (2) $\boxed{\rho = R}$

Therefore, the resistivity or the specific resistance of the material of a conductor is the resistance offered by a wire of this material of unit length and unit area of cross section.

Unit of Resistivity :- From eqn (2) SI unit of ρ = ohm $\frac{m^2}{m}$

$\therefore$ SI unit of ρ = ohm meter (Ωm)

Dimensional Formula :- $[\rho] = [R]\frac{[A]}{[l]}$

$$[\rho] = [ML^2T^{-3}A^{-2}]\frac{[L^2]}{[L]} = [ML^3T^{-3}A^{-2}]$$

MATERIAL	RESISTIVITY AT 0°C (Ωm)	MATERIAL	RESISTIVITY AT 0°C (Ωm)
Conductors		Semi-Conductors	
Silver	1.6×10^{-8}	Carbon [Graphite]	3.5×10^{-5}
Copper	1.7×10^{-8}	Germanium	0.46
Aluminium	2.7×10^{-8}	Silicon	2300
Iron	10×10^{-8}		
Alloys		Insulators	
Manganin	48×10^{-8}	Glass	$10^{10} - 10^{14}$
Constantan	49×10^{-8}	Hard Rubber	$10^{13} - 10^{16}$
Nichrome	100×10^{-8}	NaCl	$\approx 10^{14}$
		Fused Quartz	$\approx 10^{16}$

$$\boxed{\rho_{cond.} < \rho_{alloy} < \rho_{semicond.} < \rho_{insulator}}$$

The free e's in alloys find a disordered arrangement of constituent ions. Due to this e's are scattered frequently and randomly.

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NOTE:→ Alloys of Metals have a greater resistivity than their constituent metals :→

In an alloy eg. Nichrome (Ni-Cr alloy), Ni^{2+} & Cr^{3+} ions have different charge & size. They occupy random locations relative to each other, though their ionic sites form a regular crystalline lattice. An electron, therefore, passes through a very random medium & is very frequently deflected. So there is a small relaxation time and hence large resistivity. $\left(\text{As } \rho \propto \frac{1}{\tau}\right)$

Q⇒ A wire of resistance R is drawn out so that its length becomes n times of its original length. Calculate its new resistance.

Solⁿ⇒ $R_1 = R = \frac{\rho l}{A}$ —— ①

$l_2 = n l_1 = n l$

as volume will be the same

$\therefore A_1 l_1 = A_2 l_2$

$A l = A_2 n l$

$A_2 = \frac{A}{n}$

$R_2 = \frac{\rho l_2}{A_2} = \frac{\rho n l}{A/n}$

$R_2 = n^2 \frac{\rho l}{A} \Rightarrow \boxed{R_2 = n^2 R}$

Q⇒ A wire of resistance R is drawn out so that its radius of cross section becomes 1/n times of its original value. Calculate its new resistance.

Solⁿ⇒ $R_1 = R = \frac{\rho l}{A} = \frac{\rho l}{\pi r^2}$ $[\because A = \pi r^2]$

$r_2 = \frac{1}{n} r_1 = \frac{r}{n} \Rightarrow A_2 = \frac{\pi r^2}{n^2} = \frac{A}{n^2}$

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as volume will be the same

$$\therefore \quad A_1 l_1 = A_2 l_2$$

$$\pi r^2 l_1 = \pi \frac{r^2}{n^2} l_2$$

$$l_2 = n^2 l$$

$$R_2 = \frac{\rho l_2}{A_2} = \frac{\rho n^2 l}{A/n^2} = n^4 \frac{\rho l}{A} = n^4 R$$

$$\boxed{R_2 = n^4 R}$$

Q:) A cylindrical wire is stretched to increase its length by 10%. Calculate the percentage increase in resistance.

Sol^n:) Let original length of the wire be l

New length (after stretching) $l' = l + 10\%$ of l

$$l' = l + 0.1 l = 1.1 l$$

$$\frac{l'}{l} = 1.1$$

as volume remains same $\quad Al = A'l' \Rightarrow \frac{A}{A'} = \frac{l'}{l}$

$$\therefore \quad \frac{R'}{R} = \frac{l' \times A}{l \times A'} = \left(\frac{l'}{l}\right)^2 = (1.1)^2 = 1.21$$

The percentage increase in resistance,

$$\frac{R' - R}{R} \times 100 = \left(\frac{R'}{R} - 1\right) \times 100 = (1.21 - 1) \times 100 = 21\%$$

Q:) Two wires A and B of equal mass and of the same metal are taken. The diameter of the wire A is half the diameter of wire B. If the resistance of wire A is 24 Ω calculate the resistance of wire B.

Sol^n Mass of wire = Volume × density

= (area of cross section × length) × density

$$\therefore \quad m = \pi r_A^2 l_A d = \pi r_B^2 l_B d$$

$$\frac{l_B}{l_A} = \left(\frac{r_A}{r_B}\right)^2 = \left(\frac{1/2}{1}\right)^2 = \frac{1}{4} \quad \left[\because dia_A = \frac{dia_B}{2} \Rightarrow r_A = \frac{r_B}{2} \Rightarrow \left(\frac{r_A}{r_B} = \frac{1}{2}\right)\right.$$

$$\therefore \quad \frac{R_B}{R_A} = \frac{\rho \frac{l_B}{\pi r_B^2}}{\rho \frac{l_A}{\pi r_A^2}} = \frac{l_B}{l_A} \times \left(\frac{r_A}{r_B}\right)^2 = \frac{1}{4} \times \left(\frac{1}{2}\right)^2 = \frac{1}{16}$$

$$R_B = \frac{1}{16} R_A = \frac{1}{16} \times 24 = 1.5\,\Omega \text{ Ans.}$$

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EXPRESSION FOR ELECTRICAL RESISTIVITY OF A CONDUCTOR

Let $l \Rightarrow$ length of the conductor
$A \Rightarrow$ area of cross section of the conductor
$n \Rightarrow$ no. of electrons per unit volume of the conductor
$V \Rightarrow$ Potential difference applied across the conductor
$E \Rightarrow$ Electric field set up across the conductor
$R \Rightarrow$ Resistance of the conductor.

We know that,

$$v_d = \frac{eE\tau}{m} \quad \text{(in magnitude)} \text{ --- } (1)$$

and

$$I = neAv_d \text{ --- } (2)$$

$\therefore$ From eq^n (1) & (2)

$$I = \frac{ne^2AE\tau}{m}$$

or

$$\frac{V}{R} = \frac{ne^2AE\tau}{m} \quad \left[\text{as } I = \frac{V}{R}\right]$$

$$\frac{V}{\rho\frac{l}{A}} = \frac{ne^2AE\tau}{m}$$

$$\frac{EA}{\rho} = \frac{nAe^2E\tau}{m} \quad \left[\text{as } \frac{V}{l} = E\right]$$

conductivity $\rightarrow$ $\boxed{\frac{1}{\rho} = \frac{ne^2\tau}{m}}$ --- (3)

Resistivity $\rightarrow$ $\boxed{\rho = \frac{m}{ne^2\tau}}$ --- (4)

eq^n (4) gives the expression for electrical resistivity of a conductor. As, m (mass of an electron) and e (charge on an electron) both are constants, therefore.

$\boxed{(a)\ \rho \propto \frac{1}{n}}$ and $\boxed{(b)\ \rho \propto \frac{1}{\tau}}$

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Resistivity in the terms of current density :-

as $j = \frac{I}{A} = \frac{ne^2 E \tau}{m}$ —— ①

and $\rho = \frac{m}{ne^2 \tau}$ —— ②

eqⁿ ① × eqⁿ ② $j \times \rho = E$

$$\boxed{\rho = \frac{E}{j}} \text{ —— ③}$$

Resistivity in the terms of electron mobility :-

as $j = n e v_d$ and $v_d = \mu E$

hence $j = n e \mu E$

$$\boxed{\rho = \frac{1}{ne\mu}}$$

Q⇒ What is the effect of (a) length and (b) temperature of the conductor on the drift velocity of free electrons inside the conductor?

ⓐ Effect of Length on v_d ⇒ $v_d = \frac{eE\tau}{m} = \frac{e}{m}\frac{V}{l}\tau$

$$\boxed{v_d \propto \frac{1}{l}}$$

ⓑ Effect of Temperature on v_d ⇒ On increasing the temperature, the collisions of e^- with +ve metal ion will increase

∴ τ decreases

as $\boxed{v_d \propto \tau}$ ∴ v_d decreases.

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CONDUCTANCE AND CONDUCTIVITY

(a) Conductance ⇒ It is the reciprocal of resistance of a conductor.

It is denoted by 'G'.

$$\text{conductance } (G) = \frac{1}{\text{Resistance } (R)}$$

$$\boxed{G = \frac{1}{R}}$$

Units ⇒ ohm^{-1} (Ω^{-1}) or mho or Siemen (S)

Dimensions ⇒ $[M^{-1}L^{-2}T^{3}A^{2}]$

(b) Conductivity ⇒ It is the reciprocal of resistivity of a conductor.

It is denoted 'σ'

$$\text{Conductivity } (\sigma) = \frac{1}{\text{resistivity } (\rho)}$$

$$\boxed{\sigma = \frac{1}{\rho}}$$

Units ⇒ $ohm^{-1} metre^{-1}$ ($\Omega^{-1} m^{-1}$) or mho $metre^{-1}$ or Siemen $metre^{-1}$ (Sm^{-1})

Dimensions ⇒ $[\sigma] = [M^{-1}L^{-3}T^{3}A^{2}]$

OHM'S LAW IN VECTOR FORM

$$\rho = \frac{E}{j} \Rightarrow E = \rho j$$

$$E = \frac{1}{\sigma} j \quad (\sigma \text{ is conductivity})$$

$$\boxed{j = \sigma E}$$

In vector form, $\boxed{\vec{j} = \sigma \vec{E}}$ — (1)

$$\Rightarrow \boxed{\vec{j} \propto \vec{E}} \quad \text{--- (2)}$$

Thus, ohm's law in microscopic form states that -
"The current density is directly proportional to applied electric field strength and direction of $\vec{j}$ coincides with $\vec{E}$."

EFFECT OF TEMPERATURE ON RESISTIVITY

(A) **In Case of Conductors :-**

As temperature of the conductor increases, the amplitude of the vibrations of the +ve ions in the conductor also increases. Due to this the free electrons collide more frequently with the vibrating ions and thus τ decreases ∴

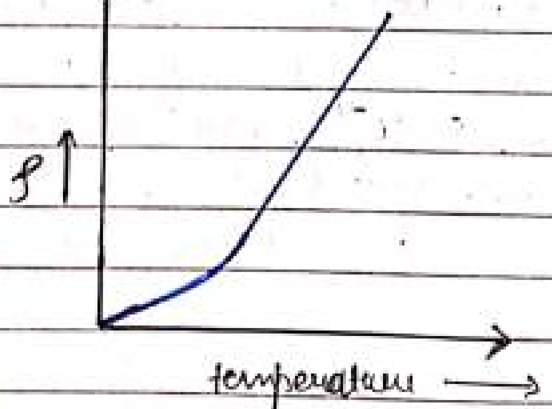
$$\text{as } \rho = \frac{m}{ne^2\tau} \quad \text{i.e., } \boxed{\rho \propto \frac{1}{\tau}}$$

$$\therefore \boxed{\rho \text{ increases}}$$

Thus, on increasing the temperature of the conductor its resistivity increases and conductivity decreases.

For pure conductors ρ increases linearly with the temperature in the temperature range around and above the room temperature.

Variation of resistivity of conductor with temp.



If ρ_0 is the resistivity of conductor at 0°C.
ρ is the " " " at θ°C
then ρ is given by

$$\boxed{\rho = \rho_0 (1 + \alpha \Delta\theta)} \quad \text{--- (1)}$$

$\alpha \Rightarrow$ constant called Temperature coefficient of Resistivity.

Similarly, as $R = \rho \frac{l}{A}$ i.e., $\boxed{R \propto \rho}$

$\therefore$ $$\boxed{R = R_0 (1+\alpha \Delta\theta)} \quad \text{— (2)}$$

$$\boxed{\alpha = \frac{R - R_0}{R_0 \Delta\theta}} \quad \text{— (4)}$$ $\Delta\theta \Rightarrow$ Rise in Temperature

$\therefore$ Temperature coefficient of resistance may be defined as change in resistance per unit original resistance per unit change in temperature.

SI unit of $\alpha \Rightarrow K^{-1}$ or $°C^{-1}$

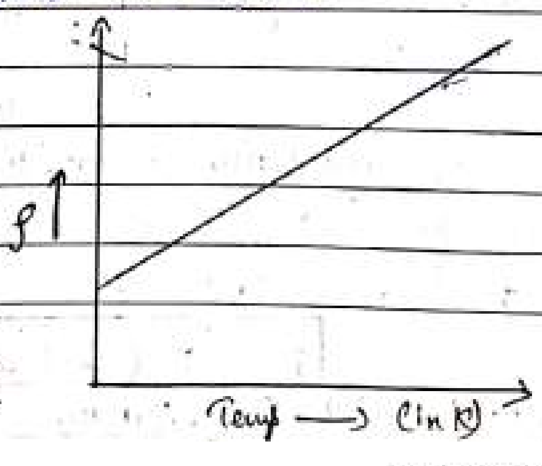
NOTE:-> (1) For metals (conductors) α is +ve and lies b/w 10^{-2} to 10^{-4} $°C^{-1}$.

(2.) At low temperature (much below 0°C) ρ increases as a higher power of temperature.

(B) In Case of Alloys :->

In this case also, ρ increases with the increase in temperature but has a very weak dependence on temperature due to small value of α.

For e.g. $\Rightarrow$ In case of Manganin α is very small ($\approx 10^{-5}$ $°C^{-1}$). For this reason only it is used as to make standard resistance coils and as a metre bridge wire and potentiometre wire.



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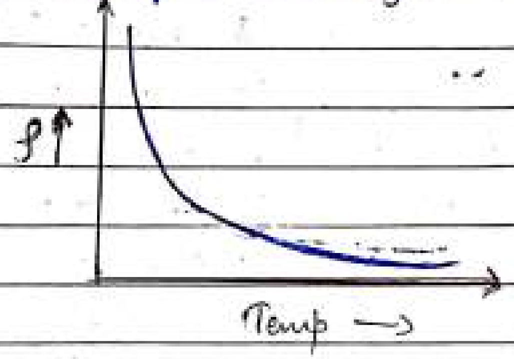
(C) In Case of Semi-Conductors and Insulators :-

In this case the value of ρ (resistivity) decreases with the increases in the temperature (for semi conductors α is negative).



(D) Electrolytes => As the temp. increases, the interionic attractions (solute-solute, solvent-solute and solvent-solvent types) decrease and also the viscous forces decrease, the ions move more freely. Hence conductivity increases or the resistivity decreases as the temp. of an electrolytic solution increases.

Uses of alloys in making standard resistors :-

1 Alloys (like constantant or manganin) are used for making standard resistance coils because of the following reasons :-

(i) They have high value of resistivity.

(ii) They have very small temperature coefficient.

(iii) They are least affected by atmospheric conditions like air, moisture, etc.

(iv) Their contact ~~with~~ potential with copper is small.

Q => The resistance of a silver wire is $1\,\Omega$ at $20^\circ C$ and $1.2\,\Omega$ at unknown temperature. If the value of α for silver is 3.8×10^{-3} $^\circ C^{-1}$. Find the unknown temperature.

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$\alpha = 3.8 \times 10^{-3}\,°C^{-1}$

Sol^n $R_0 = 1\,\Omega$ $\quad R_\theta = 1.2\,\Omega$ $\quad T_1 = 20°C$ $\quad T_2 = \theta°C$

using, $\quad R_\theta = R_0(1 + \alpha\Delta\theta) \qquad (\Delta\theta = T_2 - T_1)$

$$1.2 = 1\left[1 + 3.8 \times 10^{-3}(\theta - 20)\right]$$

$$1.2 = 1 + 3.8 \times 10^{-3}(\theta - 20)$$

$$\frac{0.2}{3.8 \times 10^{-3}} = (\theta - 20)$$

$$0.0526 \times 10^3 = \theta - 20$$

$$52.6 = \theta - 20$$

$$\boxed{\theta = 72.6°C} \text{ Ans.}$$

Q=) The resistance of a tungsten filament at 150°C is 133Ω. What will be its resistance at 500°C. Given $\alpha = 4.5 \times 10^{-3}\,°C^{-1}$.

Sol^n $R_{150} = 133\,\Omega \qquad \alpha = 4.5 \times 10^{-3}\,°C^{-1}. \qquad R_{500} = ?$

as $\quad R_t = R_0(1 + \alpha\Delta\theta) \qquad (\Delta\theta = 150 - 0)$

$$R_{150} = R_0(1 + \alpha \times 150)$$

$$133 = R_0(1 + 4.5 \times 10^{-3} \times 150) \quad \text{--- (1)}$$

$$R_{500} = R_0(1 + \alpha \times 500)$$

$$R_{500} = R_0(1 + 4.5 \times 10^{-3} \times 500) \quad \text{--- (2)}$$

Dividing (2) by (1), we get

$$\frac{R_{500}}{133} = \frac{1 + 4.5 \times 10^{-3} \times 500}{1 + 4.5 \times 10^{-3} \times 150}$$

$$\frac{R_{500}}{133} = \frac{1 + 225 \times 10^{-2}}{1 + 67.5 \times 10^{-2}} = \frac{3.25}{1.675}$$

$$R_{500} = \frac{133 \times 3.25}{1.675}$$

$$\boxed{R_{500} = 258.06\,\Omega}$$

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Q=) The resistances of iron and copper wires at 20°C are 3.9Ω and 4.1Ω respectively. At what temperature will the resistances of both be equal? Temperature coefficient of resistivity for iron is 5×10^{-3} K^{-1} and for copper it is $4.0 \times 10^{-3} K^{-1}$.

R_{Cu} at $20°C = 4.1\Omega$

R_{Fe} at $20°C = 3.9\Omega$

Solⁿ let resistance of iron wire at t°C = Resistance of copper wire at t°C.

$$\therefore \quad R_{20}[1+\alpha(t-20)] = R'_{20}[1+\alpha'(t-20)]$$

$$3.9[1+5\times10^{-3}(t-20)] = 4.1[1+4\times10^{-3}(t-20)]$$

$$(3.9\times5 - 4.1\times4)\times10^{-3}\times(t-20) = 4.1-3.9$$

$$t-20 = \frac{0.2}{3.1\times10^{-3}} = 64.5$$

$$t = 64.5+20 = 84.5°C$$

Q=) The resistance of a conductor at 20°C is 3.15Ω and at 100°C 3.75Ω. Determine the value of α for the conductor and resistance at 0°C (R_0).

Solⁿ $R_1 = R_0(1+\alpha\Delta\theta_1)$ — ① $R_2 = R_0(1+\alpha\Delta\theta_2)$ — ②

$$\frac{Eq^n ①}{Eq^n ②} = \frac{R_1}{R_2} = \frac{1+\alpha\Delta\theta_1}{1+\alpha\Delta\theta_2}$$

$$\frac{R_1}{R_2} = \frac{1+\alpha(T_1-0)}{1+\alpha(T_2-0)}$$

$$\frac{R_1}{R_2} = \frac{1+\alpha T_1}{1+\alpha T_2}$$

Here - $T_1 = 20°C$ $R_1 = 3.15\Omega$

$T_2 = 100°C$ $R_2 = 3.75\Omega$

$$\frac{3.15}{3.75} = \frac{1+20\alpha}{1+100\alpha}$$

$$3.15 + 315\alpha = 3.75 + 75\alpha$$

$$240\alpha = 3.75 - 3.15$$

$$240\alpha = 0.6$$

$$\alpha = \frac{0.6}{240} = 0.0025°C^{-1}$$

$$R_0 = \frac{R_1}{1+\alpha t_1} = \frac{3.15}{1+0.0025\times20} = 3\Omega$$

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HEATING EFFECT OF CURRENT

'The phenomenon of the production of heat in a resistor by the flow of an electric current through it is called heating effect of current or Joule heating.'

Cause of Heating effect of current :-

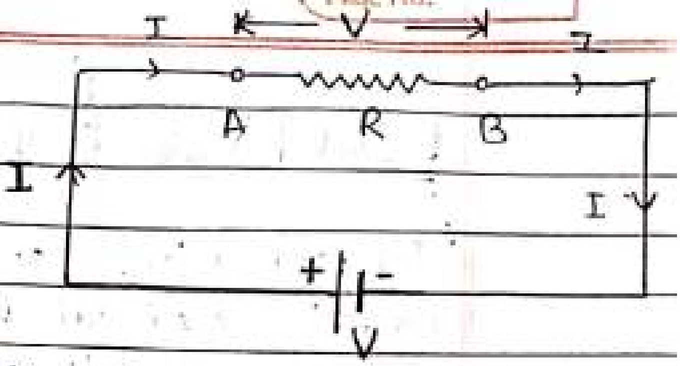
When a potential difference is applied across the ends of a conductor, its free electrons get accelerated in the opposite direction of the applied field. But the speed of the electrons does not increase beyond a constant drift speed. This is because during the course of their motion the e^-s collide frequently with the +ve metal ions. The kinetic energy gained by the e^-s during the intervals of free acceleration b/w collisions is transferred to the metal ions at the time of collision. The metal ions begin to vibrate about their mean positions more and more violently. The average kinetic energy of the ions increases. This increases the temperature of the conductor. Thus the conductor gets heated due to the flow of current. Obviously, the electrical energy supplied by the source of emf is converted into heat.

Derivation Not Important

HEAT PRODUCED BY ELECTRIC CURRENT : Joule's Law

Consider a ~~resist~~ conductor of resistance R, as shown in fig. A source of emf maintains a potential difference V b/w its ends A and B and sends a steady current I from A to B. Clearly $V_A > V_B$ and the potential difference across AB is

$$V = V_A - V_B > 0$$



The amount of charge that flows from A to B in time 't' is

$$q = It$$

as q moves through a decrease of potential of magnitude V, its potential energy decreases by the amount,

$$U = \text{Final P.E at B} - \text{Initial P.E at A}$$
$$= qV_B - qV_A = -q(V_A - V_B) = -qV < 0$$

If charges move through conductor without suffering collisions, their K.E would change so that the total energy is unchanged. By conservation of energy, the change in KE must be

$$K = -U = qV$$
$$K = It \times V = VIt > 0$$

The KE gained by the e's is shared with the metal ions and the metal ions gets heated. The amount of ~~heat~~ energy dissipated as heat in conductor in time t is

$$\boxed{\begin{array}{l} H = VIt \text{ joule} \\ H = I^2Rt \text{ joule} \\ H = \dfrac{V^2t}{R} \text{ joule} \end{array}}$$

or $H = I^2Rt$ joule (as $V = IR$)
or $H = \frac{V^2 t}{R}$ joule (as $I = \frac{V}{R}$)

The above eq^{n}s are known as Joule's Law of Heating. According to this law, the heat produced in a resistor is -

(1) $H \propto I^2$ (for a given R)
(2) $H \propto R$ (for a given I)
(3) $H \propto \frac{1}{R}$ (for a given V)
(4) $H \propto t$ (for which the current flows through the resistor)

Electric Energy and Electric Power

'The total work done/energy supplied by the source of emf in maintaining the electric current in an electrical circuit for a given time is called the electric energy consumed in the circuit.'

as $V = \frac{W}{q} \Rightarrow W = Vq$

$\therefore \boxed{W = VIt}$ (as $q = It$)

SI Unit of Electric Energy ⇒ Joule (J)

Electric Power ⇒ The rate at which work is done/energy is consumed by a source of emf in maintaining an electric current through a circuit is called its electric power.

$\therefore$ Power $= \frac{W}{t} = \frac{VIt}{t}$

$\boxed{P = VI}$

as $V = IR$ $\therefore$ $\boxed{P = I^2R}$

as $I = \frac{V}{R}$ $\therefore$ $\boxed{P = \frac{V^2}{R}}$

SI Unit of Electric Power ⇒ Joules/second = Watt (W)

or Volt Ampere (VA)

$$\boxed{1 \text{ Watt} = \frac{1 \text{ J}}{1 \text{ s}}}$$

1 Watt ⇒ The electric power of a device is said to be 1 Watt if it consumes 1 J of energy in 1 s.

or

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The electric power of a circuit is said to be 1 Watt if one ampere of current flows through it, when a potential difference of 1 V is applied across it.

Other Units of Power :-

Kilowatt (kW) = 10^3 W
Megawatt (MW) = 10^6 W
Horse Power (hp) = 1 hp = 746 W

Commercial Units of Electrical Energy

(1) Watt Hour [Wh] :- 1 Watt hour = 1 W × 1 hr
= 1 J × 3600 s
1 Wh = 3.6×10^3 J

"When a device of power 1 watt is used for 1 hour then the energy consumed by it is said to be 1 Watt Hour".

(2) Kilowatt Hour [kWh] :- 1 Kilowatt hour = 1 kW × 1 s
1 kWh = 1000 J/s × 3600 s

$$\boxed{1\ kWh = 3.6 \times 10^6\ J}$$

"When a device of power 1000 W is used for 1 hour, the energy consumed by it is said to be 1 kilowatt hour."

* Power Rating ⇒ The power rating of an electrical appliance is the electrical energy consumed per second by the appliance when connected across the marked voltage of the mains.

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CARBON RESISTORS :-> They are made from mixture of carbon black, clay and resin binder which are pressed and then moulded into cylindrical rods by heating. The rods are enclosed in a ceramic or plastic jacket. These are used in electronic circuits of radio recievers, amplifiers, etc.

Following are its advantages :->

(i) They can be made with resistance values ranging from few ohms to several millions ohms.

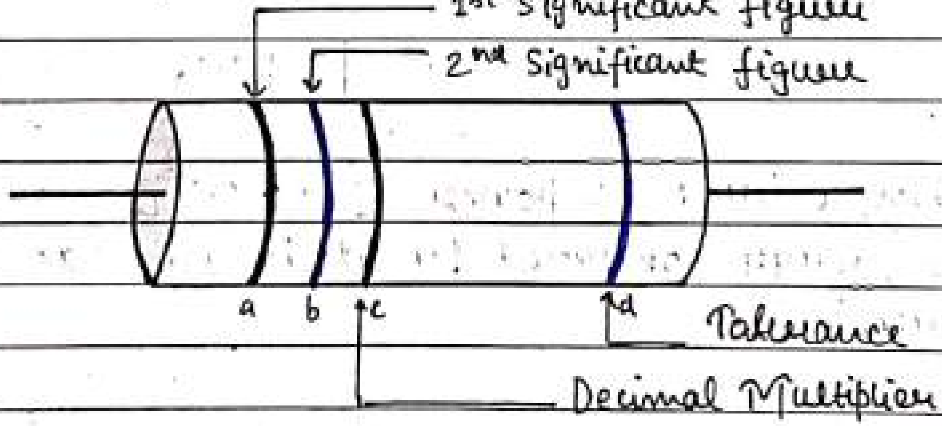
(ii) They are quite cheap and compact.

(iii) They are good enough for many purposes.

COLOUR CODE OF RESISTORS

The value of a carbon resistor is printed on its surface in the form of a colour code. It has four different coloured rings.

The first and second band gives the first and second significant figures of the resistance. The third band gives the power of 10 by which the two significant figures are to be multiplied. The fourth band indicates the tolerance (percentage reliability).



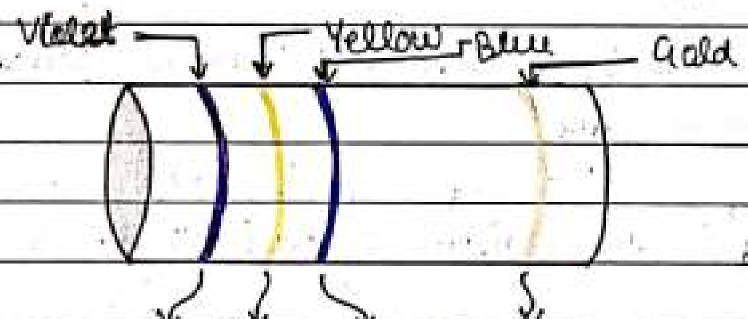
BB ROY of Great Britain had Very Good Wife

B → 0, B → 1, R → 2, O → 3, Y → 4, G → 5, B → 6, V → 7, G → 8, W → 9

The following table enables us to decode the value of carbon resistor :→

Colour	1st and 2nd Bands: Figure	3rd Band: Multipliers	4th Band: Colour	4th Band: Tolerence
Black (B)	0	10^0	Gold	5 %
Brown (B)	1	10^1	Silver	10 %
Red (R)	2	10^2	No Colour	20 %
Orange (O)	3	10^3		
Yellow (Y)	4	10^4		
Green (G)	5	10^5		
Blue (B)	6	10^6		
Violet (V)	7	10^7		
Grey (G)	8	10^8		
White (W)	9	10^9		

eg ⇒



$\boxed{7}\ \boxed{4} \times \boxed{10^6} \pm \boxed{5}\ \%$

Thus the value of resistance of ~~the~~ having the four bands in sequence as Violet, Blue, Yellow and Gold is $74 \times 10^6 \pm 5\%$

COMBINATION OF RESISTORS

Resistances can be connected in series, in parallel or their mixed combinations can be used.

(A) SERIES COMBINATION

"If a no. of resistances are connected end to end so that the same current flows through each one of them in succession, then they are said to be connected in series."

Ref. to fig. Three resistances R_1, R_2 and R_3 connected in series. When a potential difference V is applied across the combination, the same current I flows through each resistance.

By Ohm's Law, Potential Drop across the three resistances are

$$V_1 = IR_1 \quad , \quad V_2 = IR_2 \quad , \quad V_3 = IR_3$$

If V is the total potential difference, then

$$V = V_1 + V_2 + V_3$$

$$V = IR_1 + IR_2 + IR_3$$

$$V = I(R_1 + R_2 + R_3) \quad \text{——— (1)}$$

Let R_s be the equivalent resistance of the circuit.

$$\therefore \quad V = IR_s \quad \text{—— (2)}$$

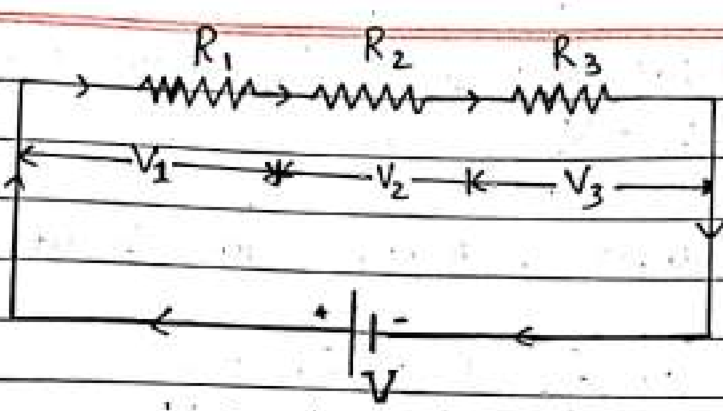
From (1) & (2), we get

$$IR_s = I(R_1 + R_2 + R_3)$$

$$\boxed{R_s = R_1 + R_2 + R_3} \quad \text{——— (3)}$$

In general for n resistances connected in series

$$\boxed{R_s = R_1 + R_2 + R_3 + \cdots + R_n}$$



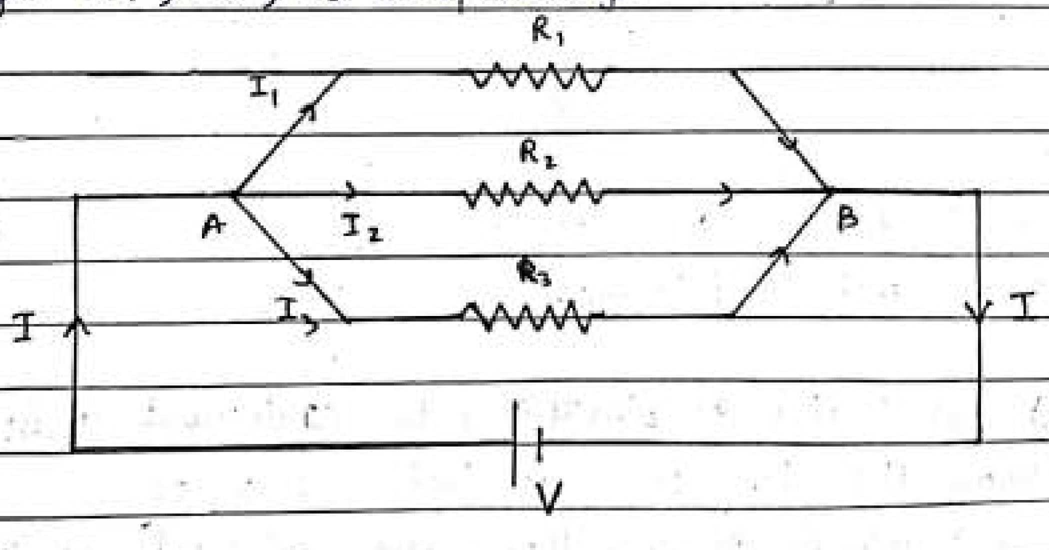
Resistance in Series.

Thus, when a no. of resistances are connected in series, their equivalent resistance is equal to the sum of the individual resistances.

(B) PARALLEL COMBINATION

'If a no. of resistances are connected between two common points so that each of them provides a separate path for current, then they are said to be connected in parallel.'

Ref. ~~to~~ fig. Three resistances R_1, R_2 and R_3 are connected in parallel b/w points A and B. Let V be the potential difference applied across the combination. Let I_1, I_2, I_3 be the ~~resistance~~ currents through R_1, R_2, R_3 respectively.



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$\because$ In parallel combination V is same across each resistor

So, $I_1 = \frac{V}{R_1}$, $I_2 = \frac{V}{R_2}$, $I_3 = \frac{V}{R_3}$ (By Ohm's Law)

Total current I through the circuit is

$$I = I_1 + I_2 + I_3$$

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$I = V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] \quad \text{——} (1)$$

If R_P is the equivalent resistance of the circuit

Then $$I = \frac{V}{R_P} \quad \text{——} (2)$$

From (1) & (2)

$$\frac{V}{R_P} = V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$$

$$\boxed{\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \quad \text{——} (3)$$

In general, for n resistances connected in parallel.

$$\boxed{\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \text{-----} \frac{1}{R_n}}$$

Thus, when a no. of resistances are connected in parallel, the reciprocal of the equivalent resistance of the parallel combination is equal to the sum of the reciprocals of the individual resistances.

NOTE: → (1) In series combination, the equivalent resistance is larger than the largest individual resistance.

(2) In parallel combination, the equivalent resistance is less than the smallest individual resistance.

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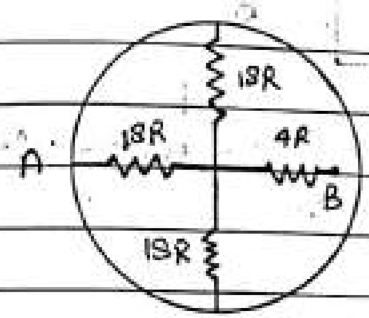
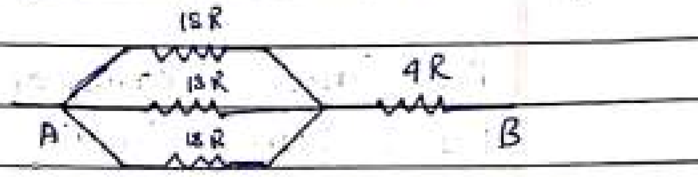
Q:) Find the equivalent resistance b/w A and B in the following :-

(a)



The given circuit can be drawn as



For the 3 resistors in parallel

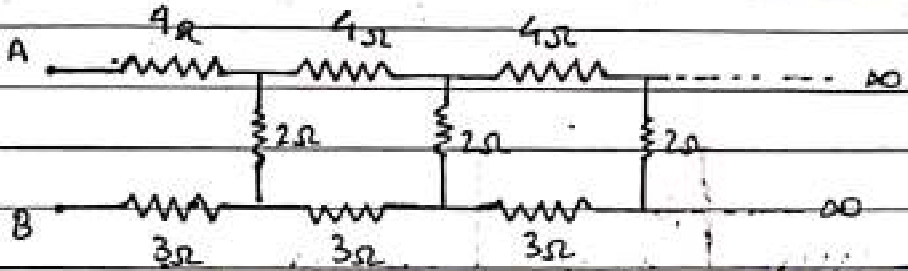
$$\frac{1}{R_p} = \frac{1}{18R} + \frac{1}{18R} + \frac{1}{18R} = \frac{3}{18R} = \frac{1}{6R}$$

$\boxed{R_p = 6R}$

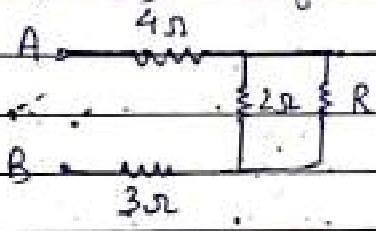
Now, 6R & 4R are in series

$\therefore$ $R_{eq} = 6R + 4R$ $\Rightarrow$ $\boxed{R_{eq} = 10R}$

(b)



Let the eq. resistance of given network be R.
As the circuit consists of infinite steps
$\therefore$ On adding or removing one step (consisting of 4Ω, 2Ω & 3Ω resistors) we get, the following circuit.



Now, as 2Ω & R are in parallel,

$\therefore$ eq. Resistance $= \frac{2R}{2+R}$

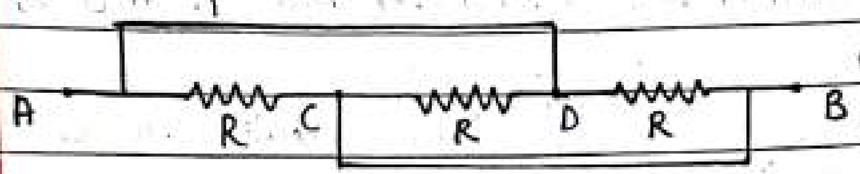
Now, $\frac{2R}{2+R}$, 4Ω and 3Ω are in series,

$\therefore$ eq. resistance (R) of circuit $= \frac{2R}{2+R} + 4 + 3 = \frac{2R}{2+R} + 7$

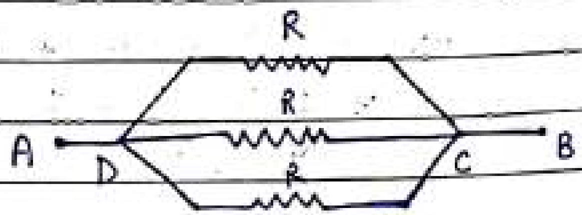
$(2+R)R = 2R + 14 + 7R$
$\cancel{2R} + R^2 = \cancel{2R} + 14 + 7R$

$\Rightarrow R^2 - 7R - 14 = 0$, on solving we get
$\boxed{R = 8.62\Omega}$

(c)



The given network of resistances can be reduced to the equivalent circuit as shown.

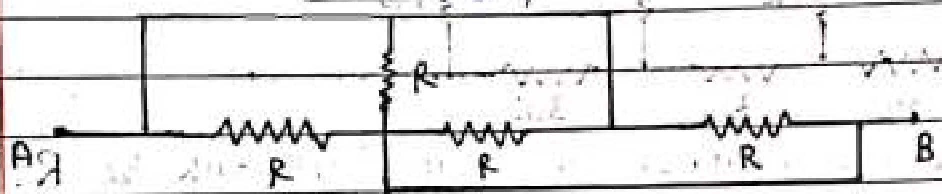


$\because$ All the three resistors are in parallel

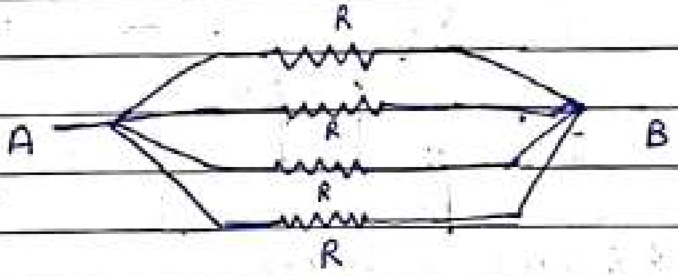
$$\therefore \frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} = \frac{3}{R}$$

$$\boxed{R_{eq} = \frac{R}{3}} \text{ Ans}$$

(d)



The given network of resistance can be reduced to the equivalent circuit as shown.



$\because$ All resistors are in ||el

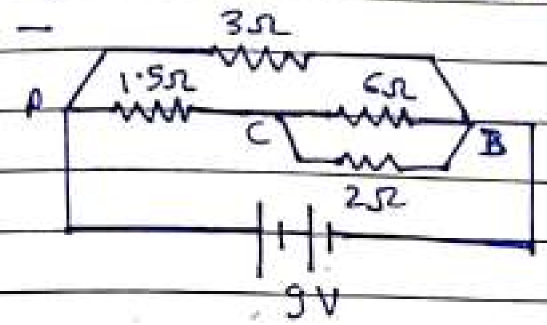
$$\therefore \frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \frac{1}{R}$$

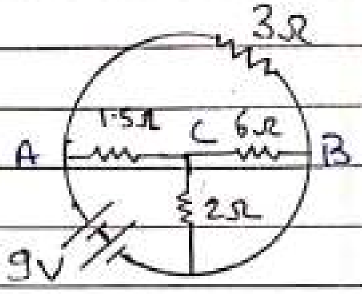
$$\frac{1}{R_{eq}} = \frac{4}{R}$$

$$\boxed{R_{eq} = \frac{R}{4}}$$

Q:) Find the current supplied by battery.

Sol^n The given circuit can be drawn as —



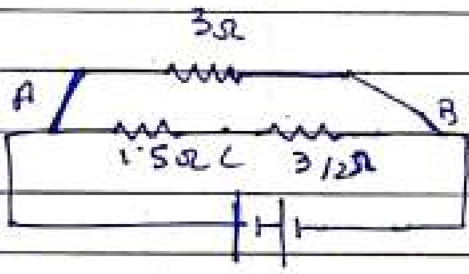


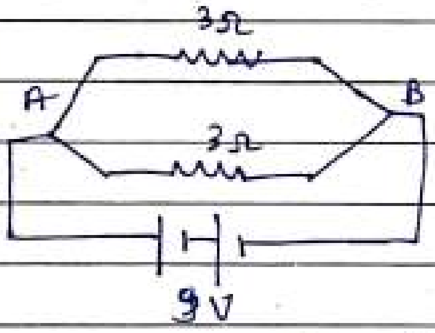
6Ω & 2Ω are in ||el $\therefore \frac{1}{R_{eq}} = \frac{1}{6} + \frac{1}{2}$

$R_{eq} = \frac{12}{8} = \frac{3}{2}$

Now, 1·5Ω and $\frac{3}{2}$Ω are in series.

$\therefore R_{eq} = 1{\cdot}5 + \frac{3}{2}\,\Omega = 3\Omega$





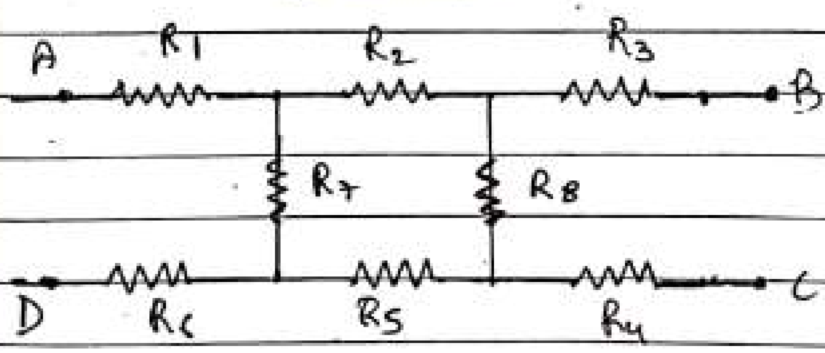
Now, 3Ω & 3Ω are in ||el

$\therefore$ ~~Req~~ $R_{eq} = \frac{3 \times 3}{3+3} = \frac{9}{6} = \frac{3}{2}\Omega$

as $I = \frac{V}{R}$ $\therefore I = \frac{9}{3/2} = 3 \times 2$

$\therefore$ $\boxed{I = 6\,A}$

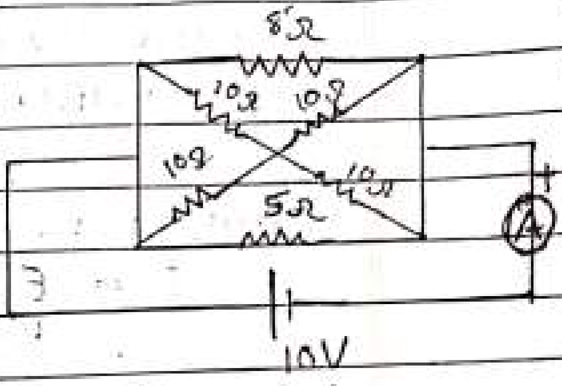
Q:)



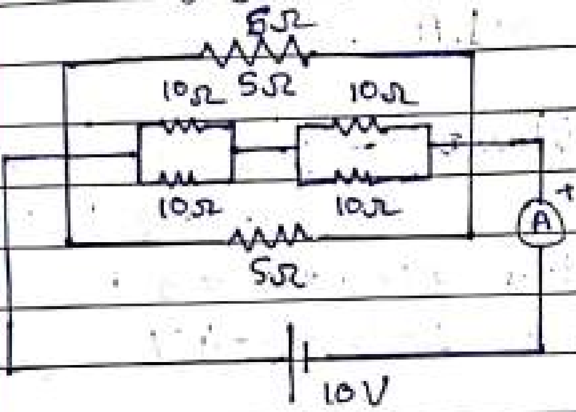
Find resistance b/w
(i) A & B
(ii) A & C
(iii) A & D

Hint:-) ignore R_7 & R_8, as current will not flow through them due to no potential difference.

Q⇒ Calculate the current shown by the ammeter A in the circuit shown.



Solⁿ⇒ The equivalent circuit is shown in the fig.

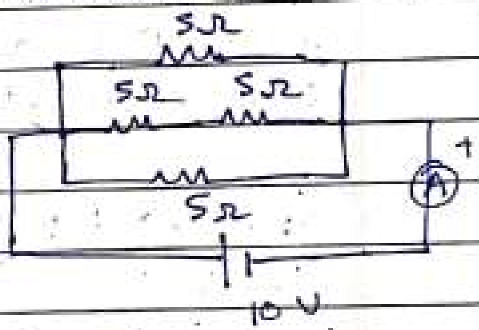


For the 10Ω resistances connected in ||el.

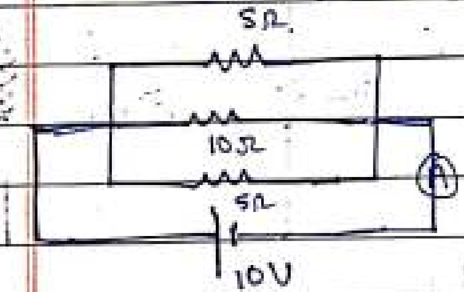
$$R_{eq} = \frac{10 \times 10}{10 + 10} = 5\Omega$$

For two such combinations connected in series

$$R_{eq} = 5 + 5 = 10\Omega$$



Now, we have resistance 5Ω, 10Ω, and 5Ω in ~~series~~ parallel



$$\frac{1}{R_{eq}} = \frac{1}{5} + \frac{1}{10} + \frac{1}{5}$$

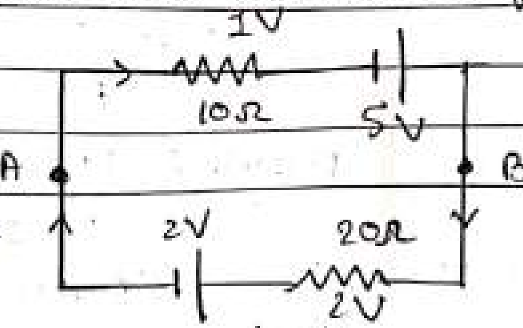
$$\frac{1}{R_{eq}} = \frac{4}{10} + \frac{1}{10} = \frac{5}{10} = \frac{1}{2}$$

$$\boxed{R_{eq} = 2\Omega}$$

Also, $V = 10\Omega$

∴ current $I = \frac{V}{R} = \frac{10}{2} = 5A$.

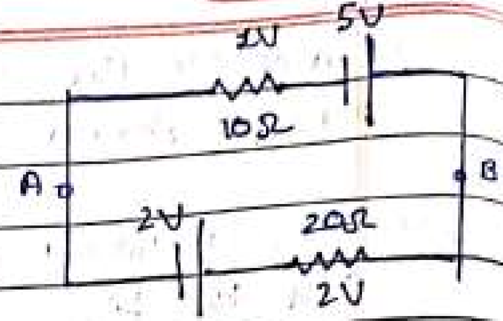
Q⇒ Find the potential difference b/w points A and B in the circuit shown in fig. Internal resistances of the cells are negligible.



Solⁿ⇒ Net emf $= 5 - 2 = 3V$

This sends a current in clockwise direction as shown in fig (next page).

Sol The point B is at higher potential than the point A.



Total resistance $= 10 + 20$
$= 30\Omega$

$\therefore$ current in circuit

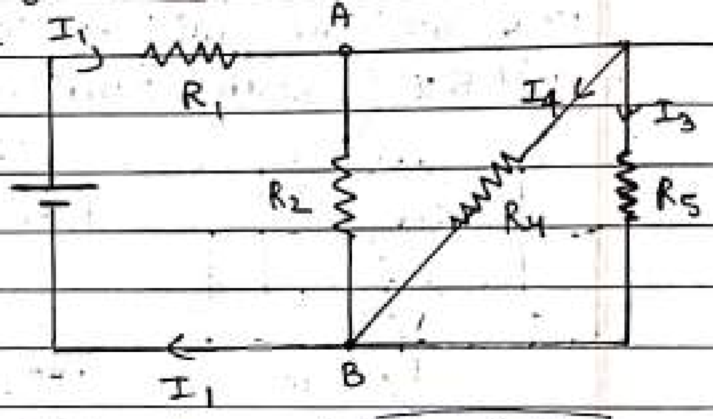
$$I = \frac{E}{R} = \frac{3}{30} = 0.1\,A$$

P.D. across 20Ω resistance $= RI = 20 \times 0.1$
$= 2V$.

P.D. across 10Ω resistance $= 10 \times 0.1 = 1V$

$$\therefore V_A - V_B = 1 - 5 = -4V$$

Q ⇒ In circuit shown in fig. $R_1 = 4\Omega$, $R_2 = R_3 = 15\Omega$, $R_4 = 30\Omega$ and $E = 10V$. Find equivalent resistance and current through each resistor.



Sol^n ⇒ R_2, R_3 & R_4 are in ||el.

$\therefore$ Their eq. resistance is R_p (R') given by

$$\frac{1}{R'} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

$$= \frac{1}{15} + \frac{1}{15} + \frac{1}{30} = \frac{5}{30} = \frac{1}{6} \qquad \therefore \boxed{R' = 6\Omega}$$

R_1 is in series with R'.

$\therefore$ eq. Resistance of the circuit $R = R_1 + R' = 4 + 6$

$R = 10\Omega$

The current I_1 is sent by E in whole circuit

$$\therefore I_1 = \frac{E}{R} = \frac{10}{10} = 1A$$

Potential drop b/w A and B

$$V = I_1 R' = 1 \times 6 = 6V$$

This is potential drop across each resistance R_2, R_3 & R_4 in ||el.

∴ currents through these resistances are -

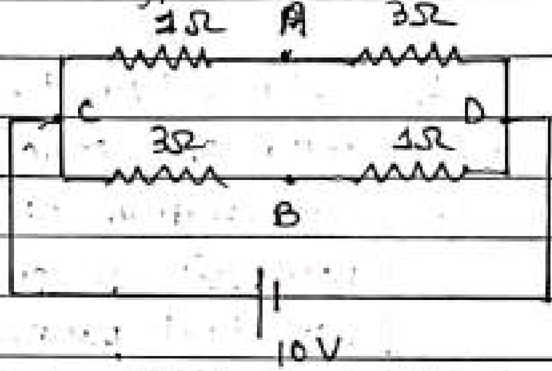
$$I_2 = \frac{V}{R_2} = \frac{6}{15} = 0.4 A$$

$$I_3 = \frac{V}{R_3} = \frac{6}{15} = 0.4 A$$

$$I_4 = \frac{V}{R_4} = \frac{6}{30} = 0.2 A.$$

Q=) A battery of emf 10 V is connected to the resistances as shown in the fig. Find the potential difference b/w the points A and B.

$1+3 = 4\Omega$
$3+1 = 4\Omega$
$\frac{1}{R} = \frac{1}{4} + \frac{1}{4}$



Solⁿ: Total resistance, $R = \frac{4 \times 4}{4+4}$

$R = 2\Omega$

current, $I = \frac{V}{R} = \frac{10}{2} = 5 A$

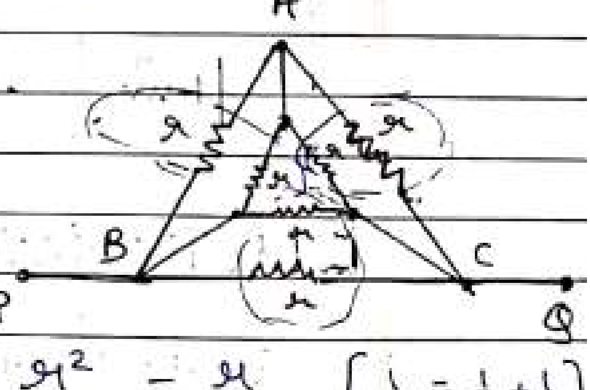
Current through each branch $= \frac{5}{2} = 2.5 A$ [as two ||el branches have same resistance 4Ω ∴ so $5A$ current is divided equally]

Now, $V_C - V_A = 2.5 \times 1 = 2.5 V$ [$V_{AC} = I_{A} \times R$, $R = 1(\Omega)$]

$V_C - V_B = 2.5 \times 3 = 7.5 V$

$$V_A - V_B = (V_C - V_B) - (V_C - V_A)$$
$$= 7.5 - 2.5 = 5 V$$

$V_A - V_B = 5V$



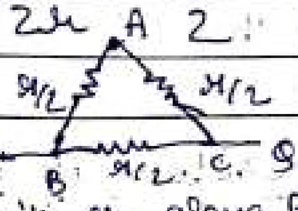
Q-) Find Req b/w P & Q.

Solⁿ The two resistances across each side of the triangle are in ||el

∴ Req of each side

$$r = \frac{r \times r}{r \times 2} = \frac{r^2}{2r} = \frac{r}{2} \quad \left[\frac{1}{R_{eq}} = \frac{1}{r} + \frac{1}{r}\right]$$

The given circuit reduces to



BA & AC are in series

$Req = \frac{r}{2} + \frac{r}{2} = r.$

This resistance is ||el with $r/2$ along BC

∴ Req $= \frac{r \times (r/2)}{r + (r/2)}$

INTERNAL RESISTANCE OF A CELL

"The resistance offered by the electrolyte of a cell to the flow of current b/w its electrodes is called Internal Resistance of the cell."

Denoted by :→ 'r'

Factors on which 'r' depends :→

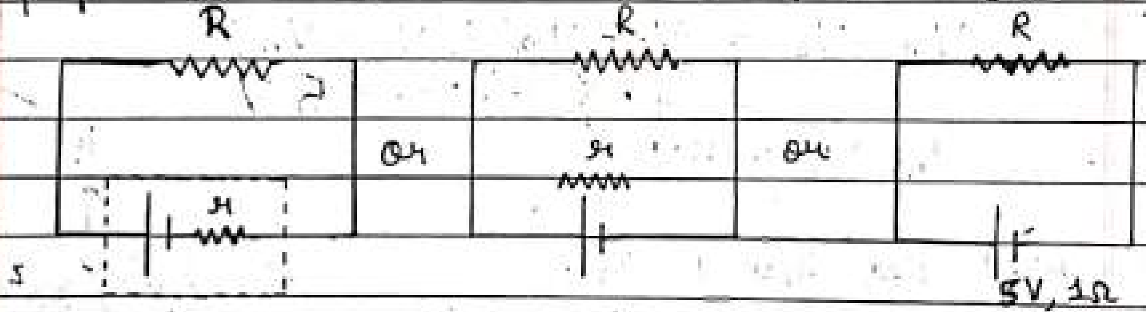
(1) Distance b/w the two electrodes ($r \propto d$)

(2) Area of electrodes dipped in the electrolyte ($r \propto \frac{1}{A}$).

(3) Temperature of electrolyte. ($r \propto \frac{1}{T}$).
For this reason only, the engines of automobiles do not start immediately when self start is operated in winters.

(4) Directly proportional to strength of electrolyte (concentration of electrolyte).

(5) How long the cell has been taken in use: For a freshly prepared cell r is less. [Nature of electrolyte].



ELECTROMOTIVE FORCE (emf) of a CELL

"It is the potential difference b/w the two terminals of the cell in an OPEN CIRCUIT (when no current is drawn from the cell)."

Denoted by ⇒ 'E' or 'ε'

The emf of a cell may also be defined as – 'work done/ energy supplied by the cell to drive a unit +ve charge around the complete circuit.'

$$\boxed{\text{emf } (\mathcal{E}) = \frac{\text{Work Done}}{\text{charge}} = \frac{W}{q}}$$

$J = V$

SI Unit ⇒ J/C or Volt.

'If an electrochemical cell supplies an energy of 1 joule for the flow of 1C of charge through the whole circuit (including the cell), then its emf is said to be 1 volt.'

TERMINAL POTENTIAL DIFFERENCE OF A CELL

'It is the potential difference b/w the two terminals of the cell in a CLOSED circuit (when current is drawn from the cell).'

Denoted By ⇒ 'V'.   SI Unit ⇒ J/C or Volt.

When current flows through the circuit, there is a drop of potential across the internal resistance of the cell. Hence, when current is drawn from the cell, the terminal potential difference of the cell is always less than the emf of the cell.

Thus,

Terminal potential difference of a cell = emf of cell − P.D. across internal resistance of the cell

$$\boxed{V = \mathcal{E} - Ir} \quad \text{--- (1)} \quad [\text{Here } V < \mathcal{E}]$$

$I = \dfrac{\text{Total emf}}{\text{Total Resistance}}$

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PAGE NO.

* **NOTE :->**

(1) If current is supplied to the cell, $V > E$.
for eg -> when cell is put under charging.
Hence, $\boxed{V = E + Ir}$ — (2)

(2) If $r = 0$, from (1) $\boxed{V = E}$

(3) If $I = 0$ (No current flows through circuit i.e. open circuit)
∴ from (1) $\boxed{V = E}$

	Electromotive Force	Potential Difference.
(1)	It is the work done by a source in taking a unit +ve charge one round the complete circuit.	It is, the amount of work done in taking a unit +ve charge from one point of a circuit to another
(2)	It is equal to the max. potential difference b/w the 2 terminals of a source when it is an open circuit.	Potential Difference may exists between any 2 points of a closed circuit.
(3)	It exists when circuit is open	It exists when circuit is closed.
(4)	It is a cause. When emf is applied in a circuit, potential difference is caused.	It is an effect
(5)	It is equal to the sum of the P.D. across all the components of a circuit including the P.D. required to send current through the cell itself.	Every circuit component has its own potential difference across its ends.
(6)	It is larger than the P.D. across any circuit element.	It is always less than emf.

(7)

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Expression for Internal Resistance of a Cell :->

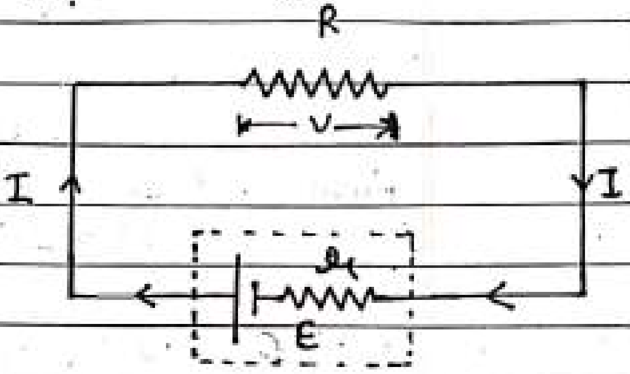
Ref. to fig.

R => External resistance

E => emf of the cell

r => internal resistance

I => current flowing in the circuit.



As $I = \frac{\text{Total emf}}{\text{Total Resistance}} \Rightarrow I = \frac{E}{R+r}$

By Ohm's Law, Potential difference across external Resistance is,

$$V = IR$$

$$V = \left(\frac{E}{R+r}\right) R$$

$$R + r = \frac{ER}{V}$$

$$\boxed{r = \frac{ER}{V} - R} \quad \text{or} \quad \boxed{r = R\left(\frac{E}{V} - 1\right)}$$

as $V = E - Ir$

$Ir = E - V$ or $\boxed{r = \frac{E - V}{I}}$

Q=> A cell of emf E and internal resistance r gives a current 0.5A with an external resistance of 12 Ω and current 0.25 A with an external resistance 25Ω. Find E and r.

Solⁿ=> **Case I** :-> $R = 12\Omega$ $\quad I = 0.5A$

$$I = \frac{\text{Total emf}}{\text{Total resistence}}$$

$$0.5 = \frac{E}{12 + r} \Rightarrow 6 + \frac{r}{2} = E$$

$$12 + r = 2E \quad \text{——— (1)}$$

Case II :- $I = 0.25\,A$, $R = 25\,\Omega$

$$0.25 = \frac{E}{25 + r}$$

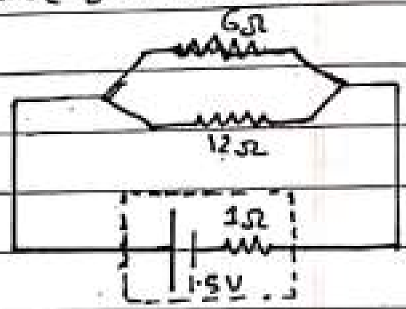
$$25 + r = 4E \quad \text{--- (2)}$$

From (1) & (2)

$$13 = 2E \Rightarrow \boxed{E = 6.5\,V}$$

Putting $E = 6.5$ in eqⁿ (1) we get $\boxed{r = 1\,\Omega}$

Q ⇒ A cell of emf 1.5 V and internal resistance $1\,\Omega$ is connected to the ||el combination of two resistances $6\,\Omega$ and $12\,\Omega$. Find the current through each resistance.



Solⁿ ⇒ $6\,\Omega$ and $12\,\Omega$ are in ||el and their equivalent resistance is in series with $1\,\Omega$

$$\therefore R_{eq} = \frac{6 \times 12}{6 + 12} + 1$$

$$R_{eq} = \frac{72}{18} + 1 \Rightarrow R_{eq} = 4 + 1$$

$$R_{eq} = 5\,\Omega$$

$$I = \frac{\text{Total emf}}{\text{Total resistance}} = \frac{1.5}{5} = 0.3\,A$$

$V = I \times (R_{eq}$ of $6\,\Omega$ and $12\,\Omega$ resistances$)$

$$V = 0.3 \times 4 = 1.2\,V$$

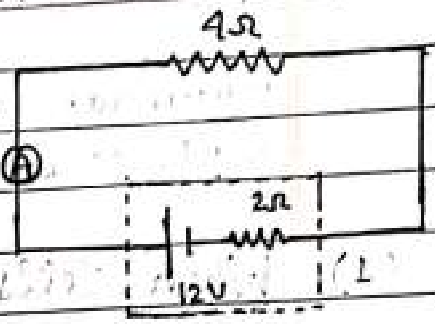
Current through $6\,\Omega$ resistor $= \frac{V}{6} = \frac{1.2}{6} = 0.2\,A$.

" " " $12\,\Omega$ " $= \frac{V}{12} = \frac{1.2}{12} = 0.1\,A$.

Q:-) In the fig. shown, find the reading of ~~apparatus~~ :-

(i) ammeter (ii) Voltmeter (when connected across cell).

(iii) Reading of Voltmeter (when connected across 4Ω resistor).



Solⁿ (i) Reading of ammeter = I

$$I = \frac{\text{Total emf}}{\text{Total resistance}}$$

$$= \frac{12}{2+4} = \frac{12}{6} = 2A$$

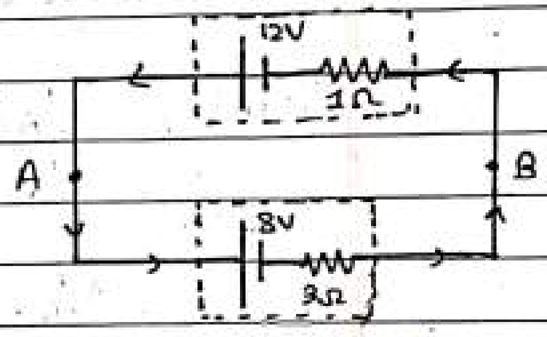
(ii) Reading of Voltmeter $= V' = E - Ir$

$$= 12 - 2 \times 2 = 8V$$

(iii) Reading of Voltmeter $= V = IR$

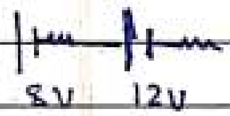
$$V = 2 \times 4 = 8V$$

Q:-) In the fig. shown, find V_{AB}.



Solⁿ Here current is drawn from 12 V cell and given to 8V cell

$$I = \frac{E_{tot}}{R_{tot}} = \frac{12 + (-8)}{3+1}$$



$$I = \frac{4}{4} = 1A$$

For cell of 12 V

$$V_{AB} = E - Ir$$

$$= 12 - (1 \times 1) = 11V$$

For cell of 8 V

$$V_{AB} = E + Ir$$

$$= 8 + (1 \times 3) = 11V$$

Grouping of Cells

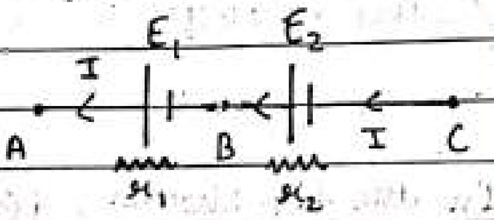
(A) Cells in Series :-

when the −ve terminal of one cell is connected to the +ve terminal of the other cell and so on, the cells are said to be connected in series.

(1) When Cells are Non-Identical :-

Consider two cells of emf E_1 and E_2 and internal resistances r_1 and r_2 are connected in series b/w points A and C. Let I be the current flowing through the series combination.

Let V_A, V_B, V_C be the potentials at points A, B and C respectively.



Clearly,
$$V_{AC} = V_A - V_C$$
$$V_{AC} = \underbrace{(V_A - V_B)}_{} + \underbrace{(V_B - V_C)}_{}$$
$$V_{AC} = (E_1 - Ir_1) + (E_2 - Ir_2)$$
$$V_{AC} = E_1 + E_2 - I(r_1 + r_2) \quad \text{——} \quad (1)$$

If E is the effective emf and r is the effective internal resistance of cells in series combination Then,
$$V_{AC} = E - Ir \quad \text{——} \quad (2)$$

comparing (1) & (2), we get
$$\boxed{E = E_1 + E_2} \quad \text{and} \quad \boxed{r = r_1 + r_2}$$

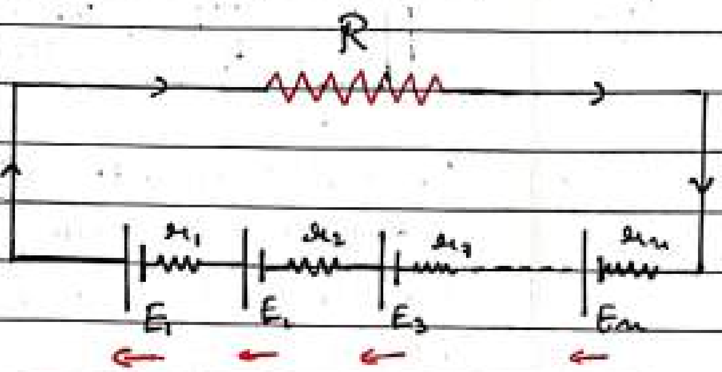
In general, for n cells in series combination equivalent emf :→

$$\boxed{E_{eq} = E_1 + E_2 + E_3 + \dots + E_n} \quad \text{— (1)}$$

equivalent internal resistance.

$$\boxed{r_{eq} = r_1 + r_2 + r_3 + \dots + r_n} \quad \text{— (2)}$$

ref. to fig.

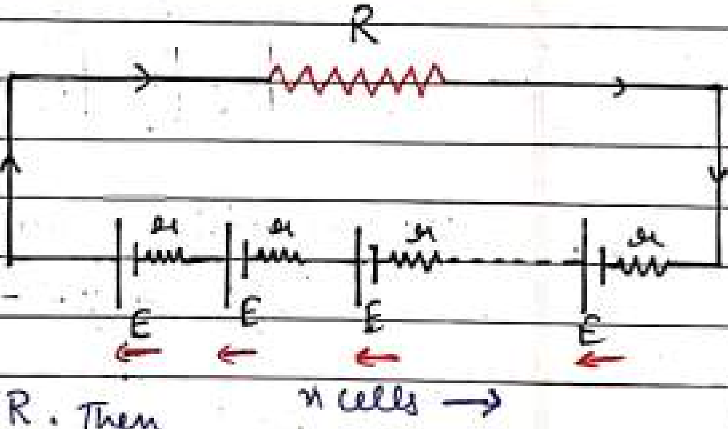


$$I = \frac{\text{Total emf}}{\text{Total resistance}}$$

$$\therefore \boxed{I = \frac{E_{eq}}{R + r_{eq}} = \frac{E_1 + E_2 + E_3 + E_4 + \dots + E_n}{R + (r_1 + r_2 + r_3 + \dots + r_n)}} \quad \text{— (3)}$$

(2) When cells are identical (same E and same r) :→

Ref. to fig



Let n identical cells each of emf E and internal resistance r are connected in series to an external resistance R. Then

$$\text{Current in Circuit} = I = \frac{\text{Total emf}}{\text{Total resistance}}$$

$$\therefore I = \frac{(E + E + E + \dots \; n \text{ times})}{R + (r + r + r + \dots \; n \text{ times})}$$

$$\boxed{I = \frac{nE}{R + nR}} \quad \text{— (4)}$$

Case I :→ If $R << nr$, then from (4)

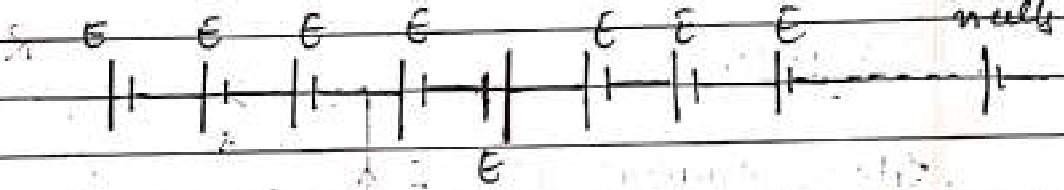
$I = \frac{nE}{nr} \Rightarrow$ $\boxed{I = \frac{E}{r} = \text{current provided by one cell}}$

Case II :→ If $R >> nr$; then from (4)

$\boxed{I = \frac{nE}{R} = n \text{ times current provided by one cell.}}$

∴ In order to get maximum current, the cells should be connected in series when the total internal resistance of the cells is negligible as compared to external resistance of the cell.

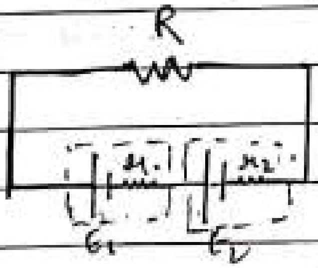
* **NOTE :→** If any of the cell is connected in the reverse order then its emf will be taken with negative (−ve) sign.



equivalent emf = (Total emf − 1 emf of opp nature) + opp. emf

$= (nE - E) - E$

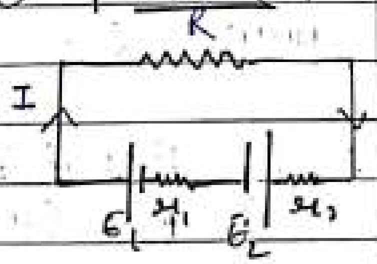
$= (n-2)E$

① 

$I = \frac{E_1 + E_2}{R + (r_1 + r_2)}$

Also, see the direction in ② & ③ of current.

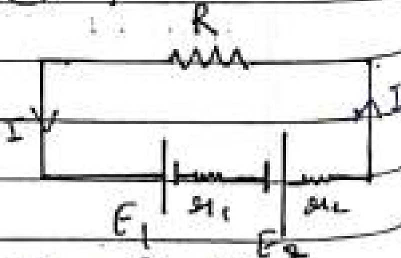
② If $E_1 > E_2$, then



$I = \frac{E_1 + (-E_2)}{R + (r_1 + r_2)}$

$I = \frac{E_1 - E_2}{R + (r_1 + r_2)}$

③ If $E_2 > E_1$, then

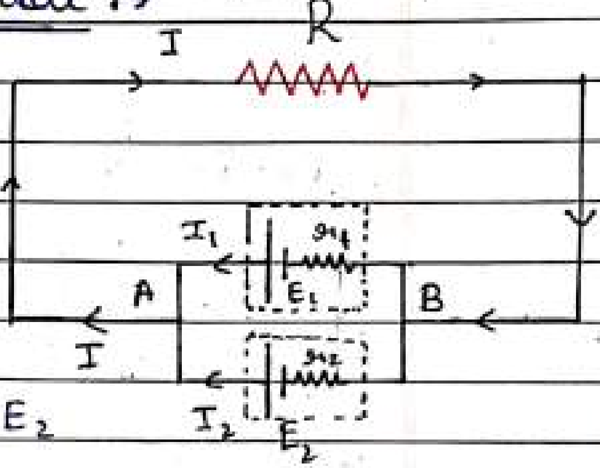


$I = \frac{-E_1 + E_2}{R + (r_1 + r_2)}$

$I = \frac{E_2 - E_1}{R + (r_1 + r_2)}$

(B) **CELLS IN PARALLEL** :→ when the +ve terminal of all cells are connected to one point and all their −ve terminals to another point, the cells are said to be connected in parallel.

(1) **When cells are Non-identical** :→



Consider two cells of emf E_1 and E_2 and internal resistance r_1 & r_2 respectively.

$I_1 \Rightarrow$ current from cell with emf E_1

$I_2 \Rightarrow$ current from cell with emf E_2

Clearly, $I = I_1 + I_2$ —— ①

$V_{AB} = V = E_1 - I_1 r_1$ —— ②

$V_{AB} = V = E_2 - I_2 r_2$ —— ③

from ② $\quad I_1 = \dfrac{E_1 - V}{r_1}$ —— ④

from ③ $\quad I_2 = \dfrac{E_2 - V}{r_2}$ —— ⑤

∴ from ①, ④ and ⑤

$$I = \frac{E_1 - V}{r_1} + \frac{E_2 - V}{r_2}$$

$$I = \frac{E_1}{r_1} + \frac{E_2}{r_2} - V\left[\frac{1}{r_1} + \frac{1}{r_2}\right]$$

$$V\left[\frac{r_1 + r_2}{r_1 r_2}\right] = \frac{E_1 r_2 + E_2 r_1}{r_1 r_2} - I$$

$$V = \frac{E_1 r_2 + E_2 r_2}{r_1 r_2}\left(\frac{r_1 r_2}{r_1 + r_2}\right) - I\left(\frac{r_1 r_2}{r_1 + r_2}\right)$$

$$\boxed{V = \left[\frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}\right] - I\left(\frac{r_1 r_2}{r_1 + r_2}\right)} \quad \text{--- (6)}$$

If E is the effective emf and r is the effective internal resistance of the parallel combination shown
Then,

$$V = E - Ir \quad \text{--- (7)}$$

comparing (6) & (7), we get

$$\boxed{E = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}} \text{ --- (8)} \quad \text{and} \quad \boxed{r = \frac{r_1 r_2}{r_1 + r_2}} \text{ --- (9)}$$

Now,

$$I = \frac{\text{Total emf}}{\text{Total resistance}}$$

$$I = \frac{\dfrac{E_1 r_2 + E_2 r_1}{r_1 + r_2}}{R + \left(\dfrac{r_1 r_2}{r_1 + r_2}\right)}$$

$$I = \frac{E_1 r_2 + E_2 r_1}{R(r_1 + r_2) + r_1 r_2} \times \frac{\cancel{r_1 + r_2}}{\cancel{r_1 + r_2}}$$

$$\boxed{I = \frac{E_1 r_2 + E_2 r_1}{R(r_1 + r_2) + r_1 r_2}} \quad \text{--- (10)}$$

NOTE:→ If there were more no. of cells, then

Equivalent emf :→ $$\boxed{E_{eq} = \frac{\Sigma (E/r_i)}{\Sigma \left(\frac{1}{r}\right)}}$$

Equivalent Resistance :→ $$\boxed{R_{eq} = R + \frac{1}{\Sigma \left(\frac{1}{r_i}\right)}}$$

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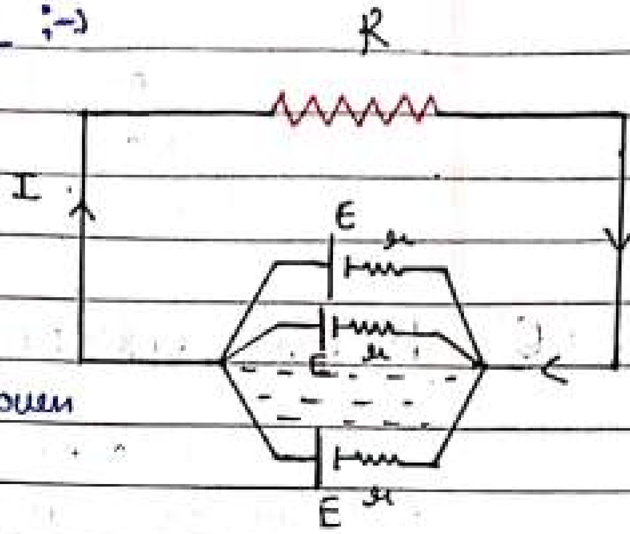
(2) When Cells are Identical :→



Let n identical cells each of emf 'E' and internal resistance 'r' are connected in parallel and then to an external resistance 'R' as shown

Here, $I = \dfrac{\text{Total emf}}{\text{Total resistance}}$

∵ All n internal resistances are connected in ||el, their equivalent resistance (say, R') is given by -

$$\frac{1}{R'} = \frac{1}{r} + \frac{1}{r} + \text{------} + n \text{ times}$$

$$R' = \frac{r}{n}$$

Total resistance $= R + R' = R + \dfrac{r}{n}$

Also, total emf of ||el combination = emf due to single cell $= E$

∴
$$\boxed{I = \frac{E}{R + \frac{r}{n}}} \quad \text{—— (1)}$$

Case I :→ If $R >> \dfrac{r}{n}$, then from (1)

$I = \dfrac{E}{R}$ = current provided by one cell

Case II :→ If $R << \dfrac{r}{n}$, then from (1)

$I = \dfrac{E}{r/n} \Rightarrow I = \dfrac{nE}{r}$ = n times of current given by one cell.

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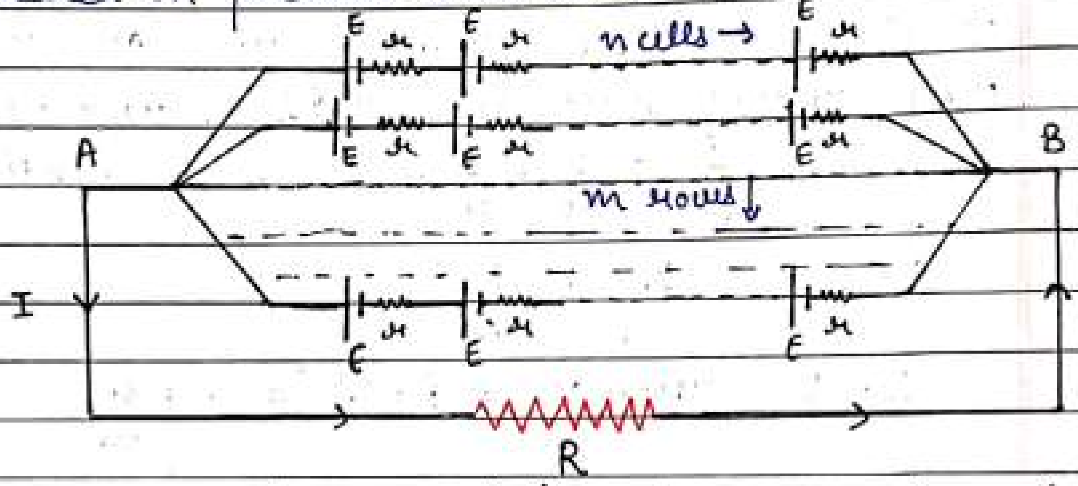
Therefore, in order to have maximum current, cells should be connected in parallel, when external resistance in the circuit is negligible as compared to total internal resistance of the cells.

(C) MIXED GROUPING OF CELLS :-

In such a combination, a certain number of identical cells are joined as series, and all such rows are then connected in parallel with each other.



Let 'n' cells, each of emf 'E' and internal resistance 'r', are connected in series in each row and 'm' such rows are connected in parallel across the external resistance 'R'.

Total no. of cells $= n \times m = N$ (say)

Total emf $= nE$

For n cells in series total internal resistance $= nr$

$\because$ there are m ||el rows of such cells

$$\therefore \frac{1}{r_{eq}} = \frac{1}{nr} + \frac{1}{nr} + \text{------ m times}$$

$$\frac{1}{r_{eq}} = \frac{m}{nr} \Rightarrow \boxed{r_{eq} = \frac{nr}{m}}$$

Total resistance $= R + \dfrac{nr}{m}$

$\therefore$ current in the circuit $(I) = \frac{\text{Total emf}}{\text{Total resistance}}$

$$I = \frac{nE}{R + \frac{nr}{m}}$$

$$\boxed{I = \frac{mnE}{mR+nr}} \quad \text{or} \quad \boxed{I = \frac{NE}{mR+nr}} \quad (1)$$

The current in the circuit will be maximum, if $mR + nR$ is minimum

$$mR + nr = (\sqrt{mR})^2 + (\sqrt{nr})^2 - 2\sqrt{mR}\sqrt{nr} + 2\sqrt{mR}\sqrt{nr}$$

$$mR + nr = \underbrace{(\sqrt{mR} - \sqrt{nr})^2}_{+\text{ve quantity}} + \underbrace{2\sqrt{mnRr}}_{+\text{ve quantity}}$$

Now clearly $mR + nr$ will be minimum if

$$\sqrt{mR} - \sqrt{nr} = 0 \quad \therefore \sqrt{mR} = \sqrt{nr}$$

$$mR = nr$$

$$\boxed{R = \frac{nr}{m}}$$

Thus, in order to have the maximum current, the cells should be connected in mixed grouping in such a manner, that the external resistance in the circuit is equal to the internal resistance of the cells.

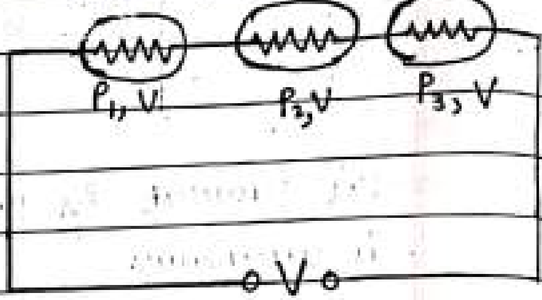
Efficiency of Source of emf

$$\text{efficiency } (\eta) = \frac{\text{output power}}{\text{input power}}$$

$$\eta = \frac{VI}{EI} = \frac{V}{E} = \frac{IR}{I(R+r)} \qquad \boxed{\eta = \frac{R}{R+r}}$$

Power Consumption in a Combination of Appliances

(1) **For Series Combination :->** consider a series of combination of 3 bulbs of powers P_1, P_2 and P_3 which work on same voltage 'V'



The Resistances of the 3 bulbs will be

$$R_1 = \frac{V^2}{P_1},\ R_2 = \frac{V^2}{P_2},\ R_3 = \frac{V^2}{P_3}$$

As bulbs are connected in series

$$\therefore\ R = R_1 + R_2 + R_3$$

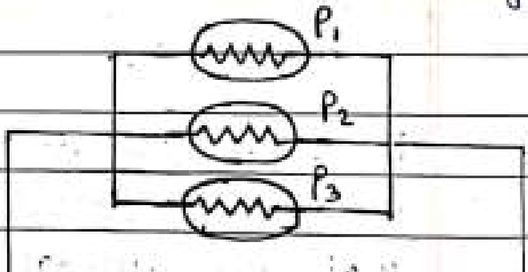
If P is the effective power of the combination

Then, $\frac{V^2}{P} = \frac{V^2}{P_1} + \frac{V^2}{P_2} + \frac{V^2}{P_3}$

$$\boxed{\frac{1}{P} = \frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3}} \quad (1)$$

(2) **For Parallel Combination :->**

Consider 3 bulbs of powers P_1, P_2 and P_3 working on same voltage V.



The resistances of 3 bulbs will be –

$$R_1 = \frac{V^2}{P_1},\ R_2 = \frac{V^2}{P_2},\ R_3 = \frac{V^2}{P_3}$$

As for ||el combination

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Let P be the effective power of the combination

$$\therefore\ \frac{P}{V^2} = \frac{P_1}{V^2} + \frac{P_2}{V^2} + \frac{P_3}{V^2}$$

$$\boxed{P = P_1 + P_2 + P_3}$$

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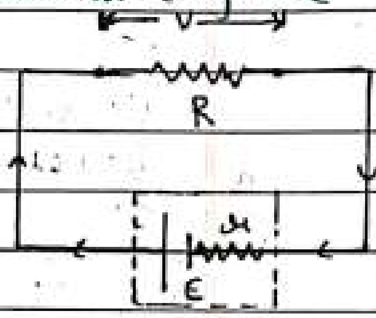
Maximum Power Theorem

It states ⇒ 'The output power of a source of emf is maximum when the external resistance in the circuit is equal to the internal resistance of the source.'



as $I = \dfrac{\text{Total emf}}{\text{Total resistance}} = \dfrac{E}{R+r}$

and $P = I^2 R$

$$P = \left(\frac{E}{R+r}\right)^2 R = \frac{E^2 R}{(R+r)^2}$$

$$P = \frac{E^2 R}{\underbrace{(R-r)^2}_{+ve} + \underbrace{4rR}_{+ve}} \quad \text{--- (1)}$$

∴ Power output will be maximum when

$R - r = 0$ or $\boxed{R = r}$

From eq.ⁿ (1) $\boxed{P_{max} = \dfrac{E^2}{4r}}$

NOTE :- When the battery is shorted, R becomes zero, therefore, power output = 0. In this case, entire power of the battery is dissipated as heat inside the battery due to its internal resistance.

Power dissipation inside the battery

$$= I^2 r = \frac{E^2}{r^2} r$$

$$\boxed{P = \frac{E^2}{r}}$$

Maximum Efficiency of Source of emf :-

For a source of emf,

Input Power = EI Output Power = VI

$\therefore$ Efficiency = $\eta = \frac{VI}{EI} = \frac{V}{E} = \frac{IR}{I(R+r)} = \frac{R}{R+r}$

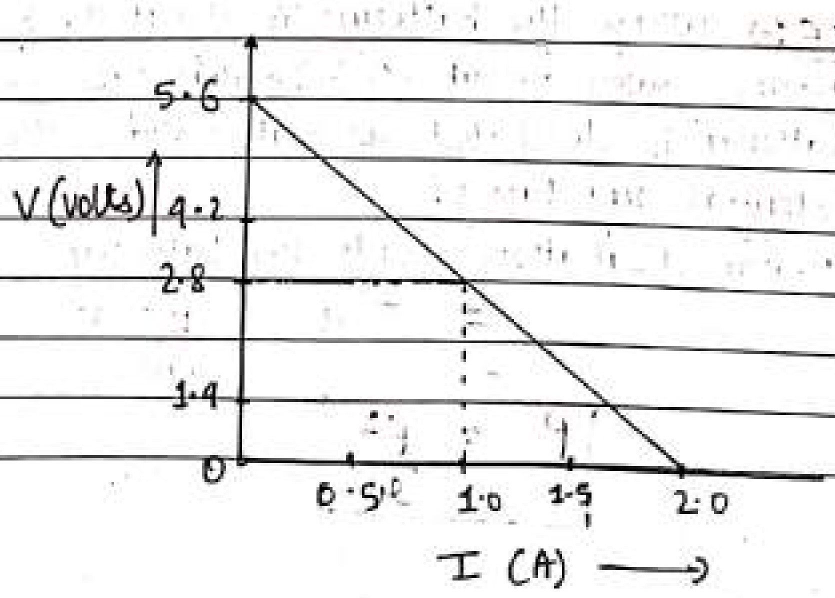
When the source of emf delivers maximum power, $R = r$

$\therefore$ $\eta = \frac{r}{r+r} = \frac{1}{2}$

$$\boxed{\eta = 50\%}$$

Q.) 4 cells of identical emf E, internal resistance r, are connected in series to a variable resistor. The following graph shows the variation of the terminal voltage of the combination with the current output.

(a) What is the emf of the each cell used?
(b) For what current from the cells, does maximum power dissipation occur in the circuit?
(c) Calculate the internal resistance.



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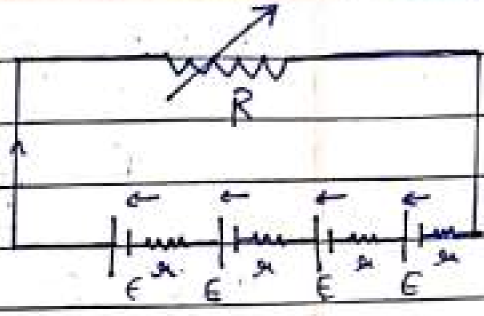
Sol^n ⇒ ref. to given graph.

When $I = 0$ then $V = 5.6\,V$



(a) as $V = E - Ir$,

If $I = 0$ then $V = E_{tot}$

Hence,

$E_{tot} = 4E \quad \therefore 4E = 5.6 \Rightarrow \boxed{E = 1.4V}$

(b) Also, from graph, when $I = 1A$, then $V = 2.8\,V$

Using $V = E_{tot} - I r_{tot}$

$$V = 4E - I(4r)$$

$$2.8 = 5.6 - 1 \times 4r$$

$$4r = 5.6 - 2.8 = 2.8$$

$$\boxed{r = 0.7\,\Omega}$$

(c) For maximum power dissipation

$$R_{ext} = \text{Total internal resistance}$$

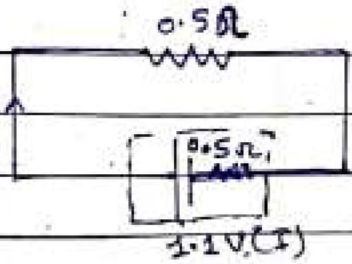
$$R = 4r = 2.8\,\Omega$$

$\therefore$ Total resistance $= R + 4r = 2.8 + 2.8 = 5.6\,\Omega$

$$I = \frac{\text{Total emf}}{\text{Total resistance}} = \frac{5.6}{5.6} = 1\,A$$

At $\boxed{I = 1\,A}$ max. power is dissipated.

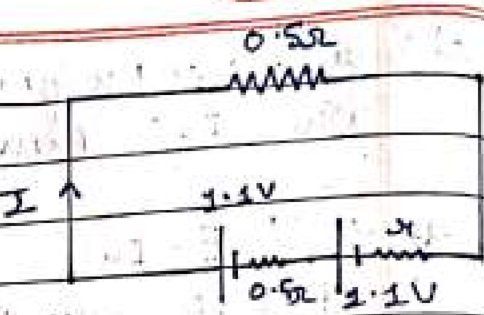
Q ⇒ A cell of emf 1.1 V and internal resistance 0.5Ω. Another cell of the same emf is connected in series but the current in the wire remains the same. Find the internal resistance of the 2nd cell.



Sol^n Case I :→ $I = \frac{\text{Total emf}}{\text{Total resistance}}$

$$I = \frac{1.1}{0.5 + 0.5} = 1.1\,A.$$

Case II :-> $I' = \dfrac{\text{Total emf}}{\text{Total resistance}}$

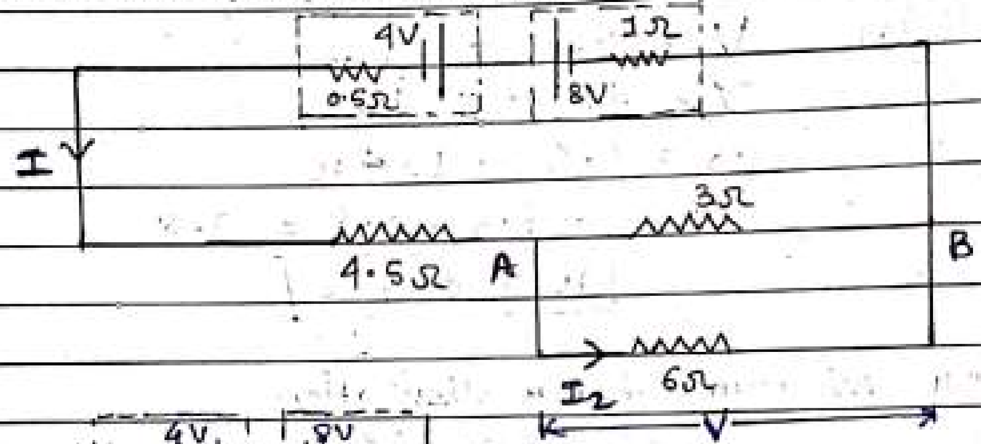


$I = \dfrac{2.2}{1+r}$

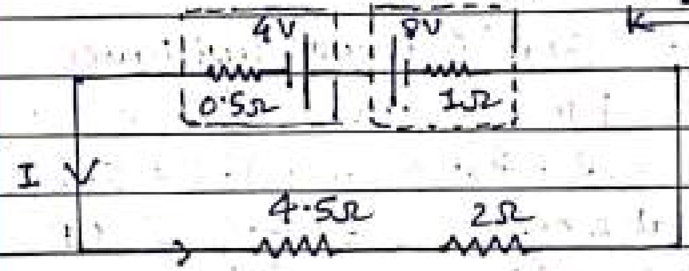
as $I = I'$ (ATQ)

$\therefore\ 1 = \dfrac{2.2}{1+r} \Rightarrow 1+r = 2 \Rightarrow \boxed{r = 1\Omega}$

Q => In the circuit diagram shown in the fig., calculate the current in each resistance.



Sol^n =>



Here the current will flow in anticlockwise directions as $8V > 4V$

Total current in the circuit

$I = \dfrac{\text{total emf}}{\text{total resistance}} = \dfrac{8+(-4)}{4.5+2+0.5+1}$

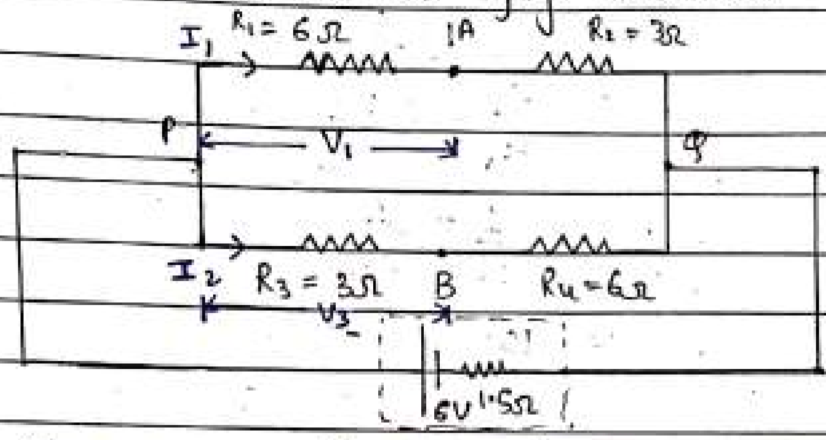
$I = \dfrac{4}{8} = \dfrac{1}{2}\,A = 0.5\,A$

& $V_{AB} = I \times R_{AB} = 0.5 \times 2 = 1V$

as $I = \dfrac{V_{AB}}{R} \Rightarrow I_1 = \dfrac{V_{AB}}{R_1} = \dfrac{1}{3}\,A$

& $I_2 = \dfrac{V_{AB}}{R_2} = \dfrac{1}{6}\,A.$

Q ⇒ Find the value of I and potential difference b/w the points A and B in the fig. shown.



$\left(\frac{3}{2} + 1.5\right)$

Sol^n ⇒ Clearly, $V_P = 6V$, Total resistance $= 6\Omega$ $\left(\frac{1}{2/9} + 1.5\right)$

∴ Total current $I = \frac{6}{6} = 1A$

Clearly, ref. to fig. $I_1 = I_2 = \frac{I}{2}$ (as both branches have equal resistance)

Now, $V_1 = I_1 R_1 = \frac{1}{2} \times 6 = 3V$

$V_A = V_P - V_1 = 6 - 3 = 3V$

then,

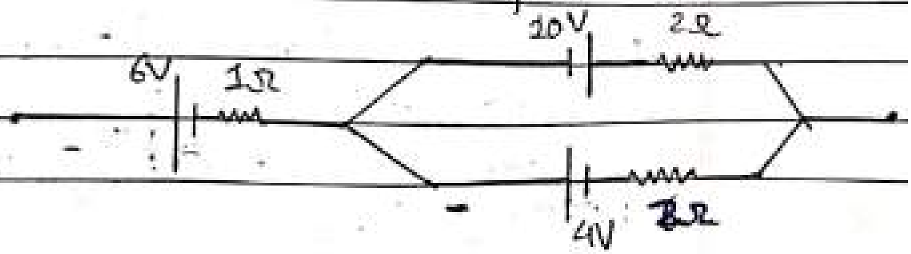
$V_3 = I_2 R = \frac{1}{2} \times 3 = 1.5V$

∴ $V_B = V_P - V_3 = 6 - 1.5 = 4.5V$

∴ Potential difference b/w the points A and B

$V_{AB} = V_B - V_A = 1.5V$ Ans.

Q ⇒ Find the emf and internal resistance of single battery which is equivalent to a combination of 3 batteries as shown below.



Solⁿ ⇒ For Parallel combination

as $E_{eq} = \dfrac{\Sigma E/r}{\Sigma 1/r}$

$= \dfrac{\dfrac{E_1}{r_1} + \dfrac{E_2}{r_2}}{\dfrac{1}{r_1} + \dfrac{1}{r_2}}$

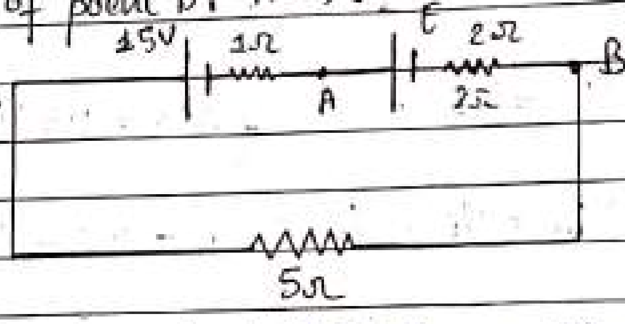
$= \dfrac{\dfrac{10}{2} + \left(\dfrac{-4}{2}\right)}{\dfrac{1}{2} + \dfrac{1}{2}}$

$E_{eq} = \dfrac{5-2}{1} = 3V$

$E_{total} = 6 + (-3) = 3V$

$R_{total} = \dfrac{1}{\frac{1}{2} + \frac{1}{2}} + 1 = 1 + 1 = 2\Omega$

Q ⇒ For what value of E, the potential of point A is equal to the potential of point B. Also, find the current in the circuit.



Solⁿ ⇒ $I = \dfrac{\text{Total emf}}{\text{Total resistance}} = \dfrac{15 + E}{1 + 2 + 5} = \dfrac{15 + E}{8}$

We have, $V_A = V_B$

$\Rightarrow V_A - V_B = 0 \quad \Rightarrow V_{AB} = 0$

But $V_{AB} = E - Ir$

$V_{AB} = E - \dfrac{(15 + E)}{8} \times 2$

$0 = \cancel{\dfrac{7E - 15}{8}}$

$\Rightarrow \dfrac{4E - 15 - E}{4} = 0 \Rightarrow 3E = 15 \quad \boxed{E = 5V}$

$I = \dfrac{20}{8}$

$I = 2.5A$

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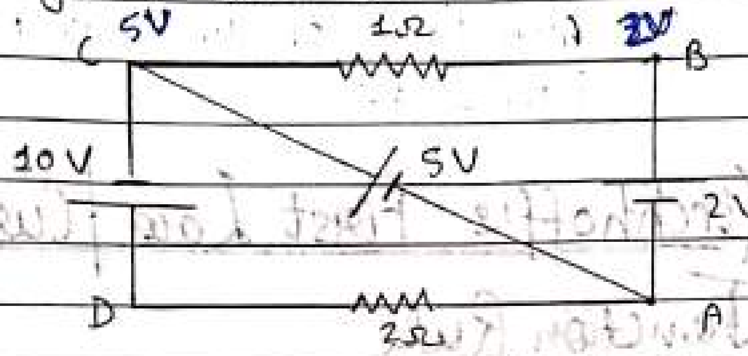
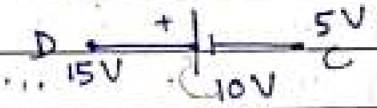
Q:) In the fig. shown, point A is connected to the ground. Find the potential of the point A, B, C and D and the current through 1Ω and 2Ω resistance.



Sol^n $V_A = 0$ (as it is connected to the ground)
$V_B = 2V$ (as it is directly connected to +ve terminal of the battery of 2V)

$V_C = 5V$ (as it is directly connected to +ve terminal of the battery of 5V)

$V_D = V_{DC} + V_C$ (as $V_{DC} = V_D - V_C$)
$V_D = +10 + 5 = 15V$



$\therefore V_{DA} = 15 - 0 = 15V$
$I = \frac{V_{DA}}{R} = \frac{15}{2} = 7.5\,A$ (D to A) ← Higher Pot. to lower Pot.

$V_{CB} = 5 - 2 = 3V$
$I' = \frac{V_{CB}}{R} = \frac{3}{1} = 3A$ [C to B] ← Higher to lower potential

Imp
KIRCHHOFF'S LAWS :->

Before understanding these laws, we define a few terms:-

(1) Electric Network => It is used for a complicated system of electrical conductors.
(2) Junction => Any point in an electric circuit where two or more conductors are joined together is a junction

(3) Loop or Mesh ⇒ Any closed conducting path in an electric network is called a loop or a mesh.

(4) Branch ⇒ Any part of the network that lies b/w two junctions.

(1) Kirchhoff's First Law / Current Law / Junction Rule :→

It states that :- 'In any electrical network, the algebraic sum of the currents meeting at a point (junction) is always zero.'

i.e. $\boxed{\Sigma I = 0}$

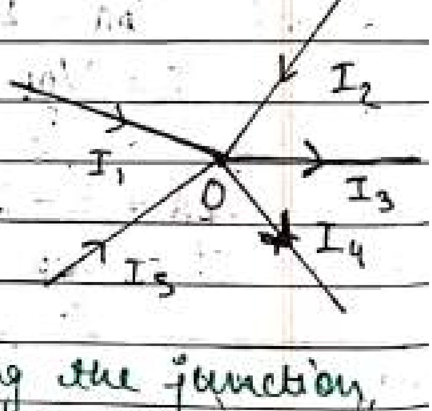
Sign Conventions :→

(i) current approaching the junction is +ve.

(ii) Current leaving the junction is −ve.

Then according to 1st law :→

$I_1 + I_2 + (-I_3) + (-I_4) + I_5 = 0$



⇒ $\boxed{I_1 + I_2 + I_5 = I_3 + I_4}$

∴ 'The sum of currents approaching the junction is equal to the sum of currents leaving the junction.'

⇒ This law is based on Law of Conservation of charge. When currents in a circuit are steady, charges cannot accumulate or originate at any point of the circuit. So whatever charge flows, towards junction in any time interval, an equal charge must flow away from that junction in the same time interval.

(2) Kirchhoff's Second Law / Kirchhoff's Voltage law / Loop Rule :-

It states that – 'In any closed path / loop / mesh in an electrical network, the algebraic sum of all potential drops and emfs is always zero.'

or

'In any closed part of an electrical circuit, the algebraic sum of the emfs is equal to the sum of the products of the resistances and the current flowing through them.'

i.e. $\boxed{\sum E = \sum IR}$ $\boxed{\sum \Delta V = 0}$

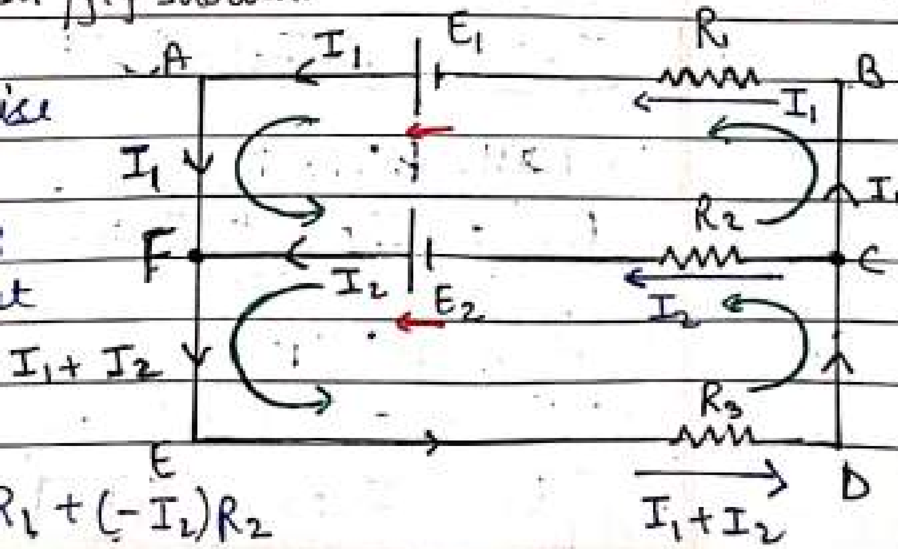
Sign Conventions :-

(i) We can take any direction (clockwise or anticlockwise) as the direction of traversal.

(ii) The emf of cell is taken as +ve if the direction of traversal is from its –ve to +ve terminal (through the electrolyte) and –ve if dir. of traversal is +ve to –ve.

← –ve to +ve

Ref. to the situation / fig shown.



Consider anticlockwise direction to be +ve

Applying Kirchhoff's law in closed part ABCFA

$\sum E = \sum IR$

$\therefore E_1 + (-E_2) = I_1 R_1 + (-I_2) R_2$

$$\boxed{E_1 - E_2 = -I_1R_1 - I_2R_2} \quad \text{—— (1)}$$

Applying Kirchhoff's law in loop FCDEF

as $\Sigma E = \Sigma IR$

$$\therefore \boxed{E_2 = I_2R_2 + (I_1 + I_2)R_3} \quad \text{—— (2)}$$

Applying Kirchhoff's law in loop ABDEA

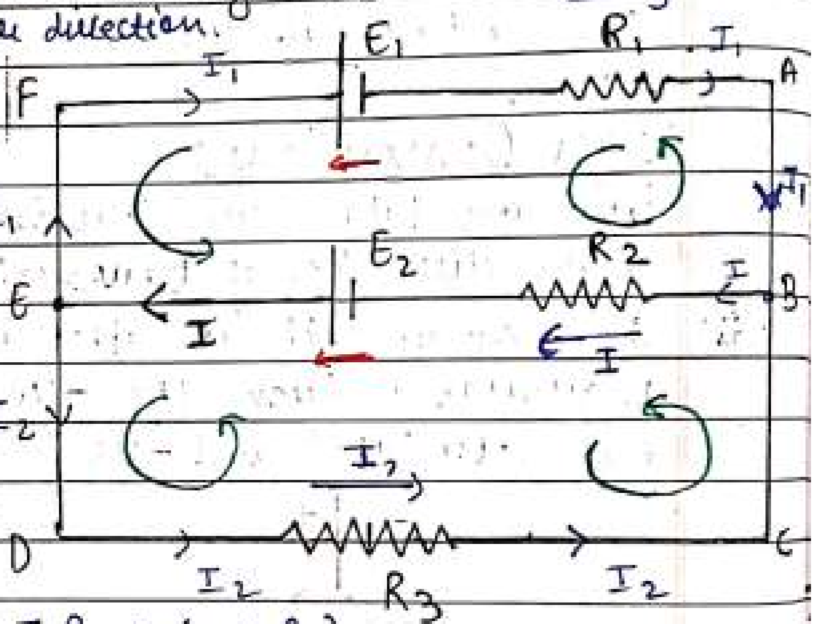
as $\Sigma E = \Sigma IR$

$$\boxed{E_1 = I_1R_1 + (I_1 + I_2)R_3} \quad \text{—— (3)}$$

Given ⇒ If I is the current through E_2

Let anticlockwise be the direction.

$I = I_1 + I_2$



Applying Kirchhoff's law in the loop ABEFA

as $\Sigma E = \Sigma IR$

$$\therefore E_1 + (-E_2) = -I_1R_1 + (-IR_2)$$

$$E_1 - E_2 = -I_1R_1 - IR_2$$

$$\boxed{E_2 - E_1 = I_1R_1 + IR_2} \quad \text{—— (1)}$$

In Mesh BCDEB,

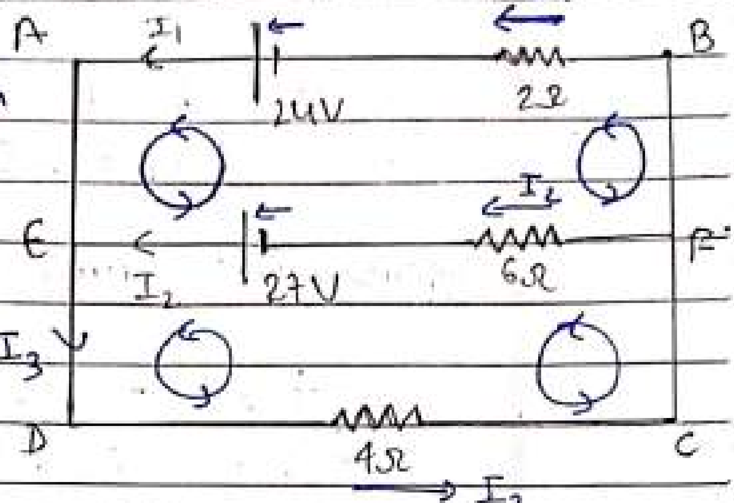
$$\boxed{E_2 = IR_2 + I_2R_3} \quad \text{—— (2)}$$

In Mesh ACDFA

$$E_1 = -I_1R_1 + I_2R_3$$

$$\boxed{E_1 = I_2R_3 - I_1R_1} \quad \text{—— (3)}$$

Q:- Find the values of I_1, I_2 & I_3.
Take anticlockwise direction the +ve



Sol^n Applying Loop Rule in mesh ABFEA $\Sigma E = \Sigma IR$

$\cancel{24-27}$

$24+(-27) = 2I_1 + (-6I_2)$

$3 = 6I_2 - 2I_1$ — (1)

Applying loop rule in mesh FEDCF.

$27 = 6I_2 + 4I_3$ — (2)

Applying loop rule in ABCDA

$24 = 2I_1 + 4I_3$

$12 = I_1 + 2I_3$ — (3)

Also, $I_3 = I_1 + I_2$

∴ eq^n (3) becomes $12 = I_1 + 2I_1 + 2I_2$

$12 = 3I_1 + 2I_2$ — (4)

Eq^n (1) − 3 × eq^n (4)

$3 - 36 = 6\cancel{I_2} - 2I_1 - 9I_1 - 6\cancel{I_2}$

$-33 = -11I_1$

$\boxed{I_1 = 3A}$

Put in eq^n (1) $3 = 6I_2 - 6$

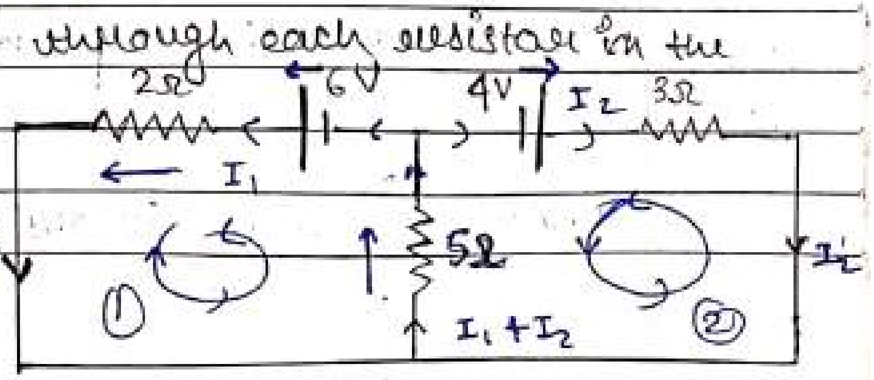
$6I_2 = 9 \Rightarrow I_2 = \frac{3}{2} = 1.5A$

$I_3 = I_1 + I_2$

$= 3 + 1.5 = 4.5A$.

Q:- Find the current through each resistor in the fig. shown.

Consider anticlockwise direction to be +ve



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Sol^n:) Applying loop rule in mesh 1

$$6 = 2I_1 + 5(I_1 + I_2)$$
$$6 = 7I_1 + 5I_2 \quad \text{--- (1)}$$

Applying loop rule in mesh 2

$$-4 = -3I_2 + (-5)(I_1 + I_2)$$
$$4 = 3I_2 + 5I_1 + 5I_2$$
$$4 = 8I_2 + 5I_1 \quad \text{--- (2)}$$

$5 \times eq^n$ (2) $-$ $8 \times eq^n$ (1)

$$20 - 48 = \cancel{40I_2} + 25I_1 - 56I_1 - \cancel{40I_2}$$
$$-28 = -31I_1$$
$$I_1 = \frac{28}{31} A$$

Put in eq^n (1)

$$6 = \frac{7 \times 28}{31} + 5I_2$$
$$5I_2 = 6 - \frac{196}{31}$$
$$I_2 = \frac{-10}{31} \times \frac{1}{5} \Rightarrow I_2 = \frac{-2}{31} A$$

Current through 2Ω resistor $= I_1 = \frac{28}{31} A = 0.903 A$

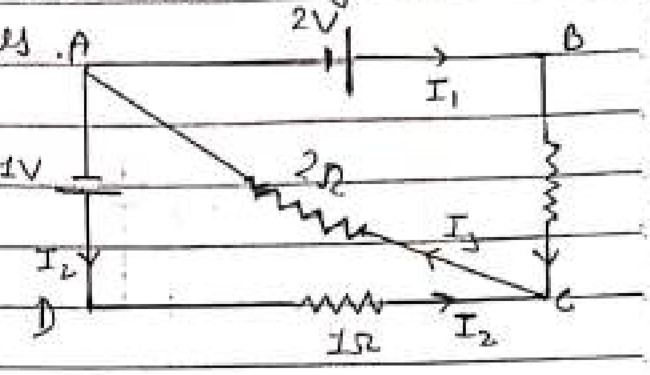
Current through 3Ω resistor $= I_2 = \frac{-2}{31} A = -0.064 A$

$-$ve sign shows that current flows in the opposite direction to the assumed direction.

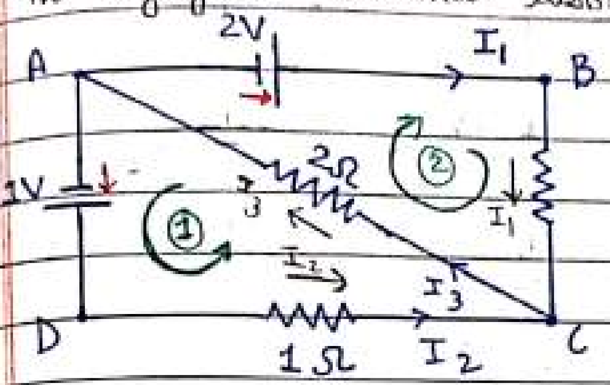
Current ~~flow~~ through 5Ω resistor $= I_1 + I_2 = \frac{28}{31} - \frac{2}{31}$

$$= \frac{26}{31} A = 0.838 A$$

Q→ Calculate the currents I_1, I_2 and I_3 in the given circuit. The negligible internal resistances.



Sol^n →



Clearly, $I_3 = I_1 + I_2$ —— (i)

Applying Kirchoff's Loop Rule in mesh (1)

$1 = I_2 + 2I_3$

$1 = I_2 + 2(I_1 + I_2)$

$2I_1 + 3I_2 = 1$ —— (ii)

Applying Kirchoff's Loop Rule in mesh (2)

$2 = I_1 + 2I_3$

$2 = I_1 + 2(I_1 + I_2)$

$3I_1 + 2I_2 = 2$ —— (iii)

3 × eq^n (ii) − 2 × eq^n (iii)

$\cancel{6I_1} + 9I_2 - \cancel{6I_1} - 4I_2 = 3 - 4$

$5I_2 = -1$

$I_2 = -\frac{1}{5}$ ⇒ $\boxed{I_2 = -0.2A}$

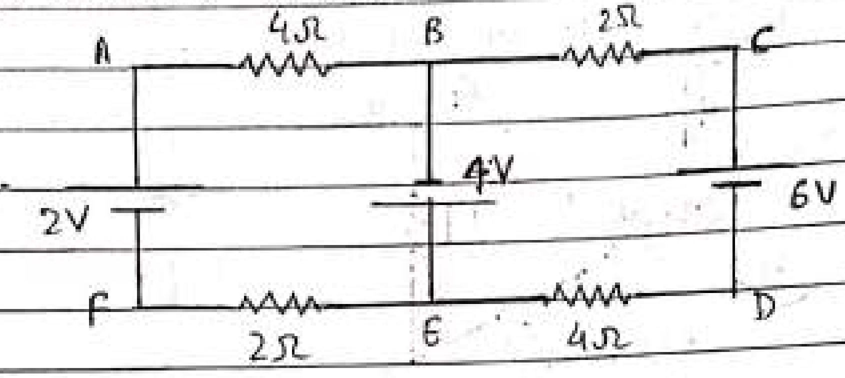
from eq^n (ii) $2I_1 = 1 - 3I_2$

$2I_1 = 1 + 3 \times 0.2$

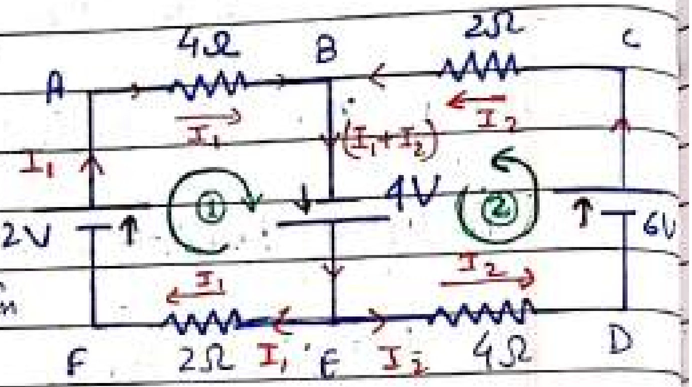
$I_1 = \frac{1.6}{2}$ ⇒ $\boxed{I_1 = 0.8A}$

∴ $I_3 = I_1 + I_2$ ⇒ $\boxed{I_3 = 0.6A}$

Q => Find the current in each branch of the circuit shown.



Sol^n => Distribution of current in various branches is shown in the fig.



Applying Kirchoff's Loop Rule in loop ABEFA

$$2+4 = 4I_1 + 2I_1$$

$$6I_1 = 6 \Rightarrow \boxed{I_1 = 1A}$$

Applying Kirchoff's Loop Rule in loop CDEBC

$$4+6 = 2I_2 + 4I_2$$

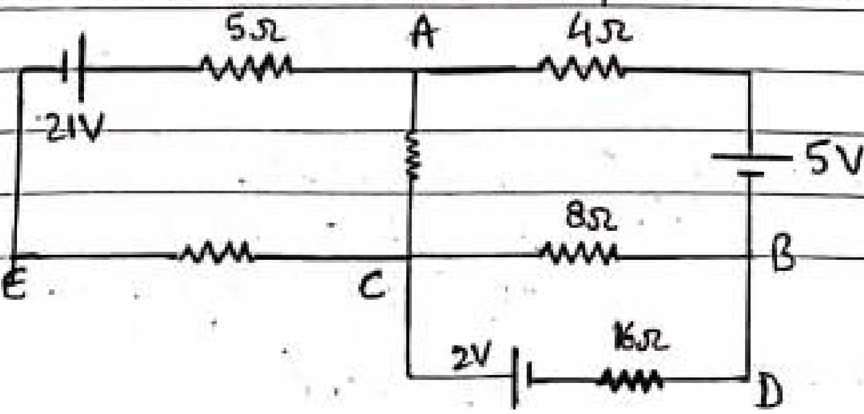
$$10 = 6I_2 \Rightarrow \boxed{I_2 = \frac{5}{2}A}$$

∴ Current in branch AB, FE and AF = $I_1 = 1A$

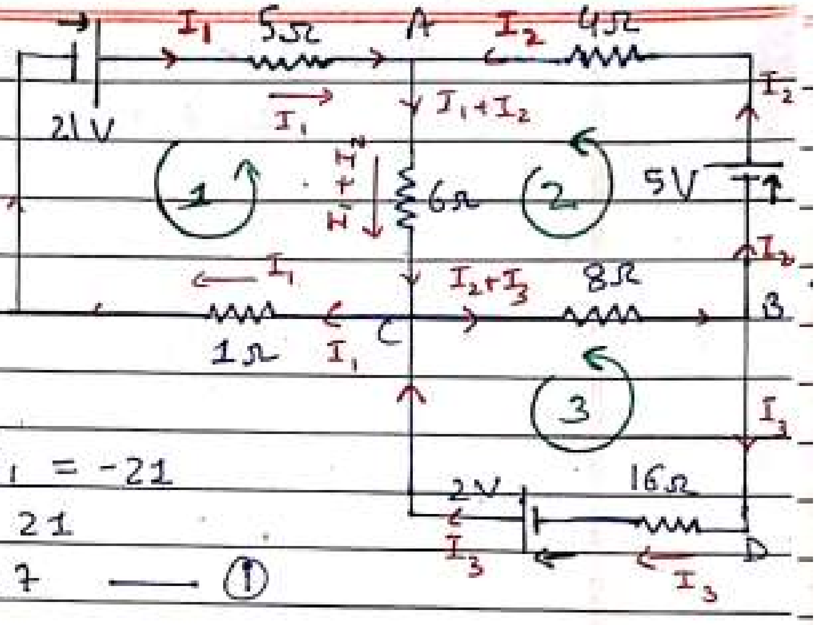
" " " BC, CD and DE = $I_2 = \frac{5}{3}A = 1.67A$

" " " BE = $I_1 + I_2 = 1 + \frac{5}{3} = \frac{8}{3}A = 2.67A$

Q => Find the current in each branch of the circuit in fig shown



Solⁿ ⇒ Distribution of current in various branches is shown in fig.



Applying Kirchoff's Loop Rule in Loop ①

$$-5I_1 - 6(I_1+I_2) - I_1 = -21$$
$$12I_1 + 6I_2 = 21$$
$$4I_1 + 2I_2 = 7 \text{ --- (i)}$$

Applying Kirchoff's loop rule in loop ②

$$4I_2 + 6(I_1+I_2) + 8(I_2+I_3) = 5$$
$$6I_1 + 18I_2 + 8I_3 = 5 \text{ --- (ii)}$$

Applying Kirchoff's loop rule in loop ③

$$-16I_3 - 8(I_2+I_3) = -2$$
$$24I_3 + 8I_2 = 2$$
$$12I_3 + 4I_2 = 1 \text{ --- (iii)}$$

(3× eqⁿ (i)) − (2× eqⁿ (ii))

$$(12I_1 + 6I_2) - (12I_1 + 36I_2 + 16I_3) = 21 - 10$$
$$-30I_2 - 16I_3 = 11 \text{ --- (iv)}$$

(4× eqⁿ (iii) + 3× eqⁿ (iv))

$$(48I_3 + 16I_2) + (-90I_2 - 48I_3) = 4 + 33$$
$$-74I_2 = 37$$
$$\boxed{I_2 = -\frac{1}{2}\text{ A}}$$

Put $I_2 = -\frac{1}{2}$ in eqⁿ (i) & eqⁿ (iii)

$$4I_1 - 1 = 7$$
$$\boxed{I_1 = 2\text{ A}}$$

$$12I_3 - 2 = 1$$
$$\boxed{I_3 = \frac{1}{4}\text{ A}}$$

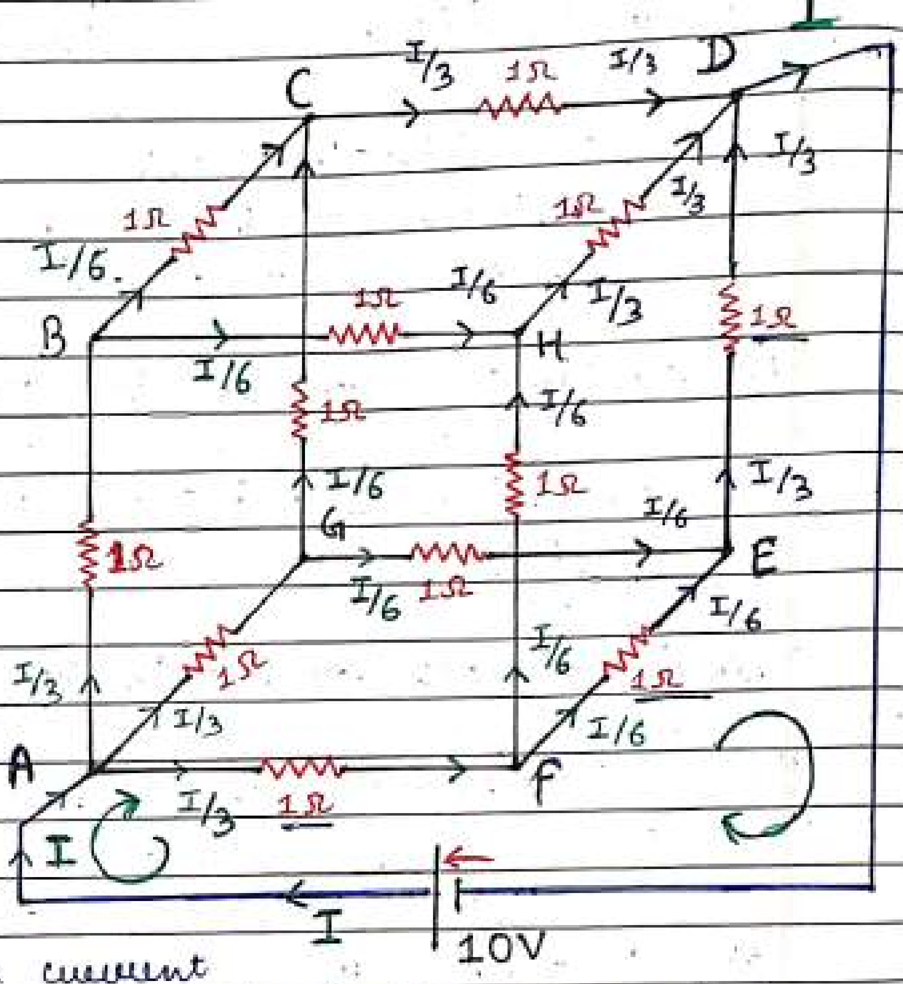
∴ current in various branches is

$I_{CEA} = I_1 = 2A = I_{EA}$ | $I_{AC} = I_1 + I_2 = \frac{3}{2} A$

$I_{BA} = I_2 = -\frac{1}{2} A$ | $I_{CB} = I_2 + I_3 = -\frac{1}{2} A$

$I_{DC} = I_3 = \frac{1}{4} A$

NCERT Q ⇒ 12 wires, each of resistance 1 Ω are connected in the form of a skeleton cube. Find the equivalent resistance of the cube, when the current enters at one corner and leaves at the diagonally opposite corner of the cube.



Solⁿ ⇒ Let I be the current through the cell. The distribution of current in various resistors is shown in the fig. [The distribution is as per the symmetry].

Applying Kirchoff's loop rule in loop AFEDA

$$\Sigma E = \Sigma I \cdot R$$

$$10 = \left(1 \times \frac{I}{3}\right) + \left(1 \times \frac{I}{6}\right) + \left(1 \times \frac{I}{3}\right)$$

$$10 = \frac{2I}{3} + \frac{I}{6} = \frac{4I + I}{6}$$

$$10 = \frac{5I}{6} \Rightarrow I = \frac{10 \times 6}{5} = 2 \times 6 = 12A$$

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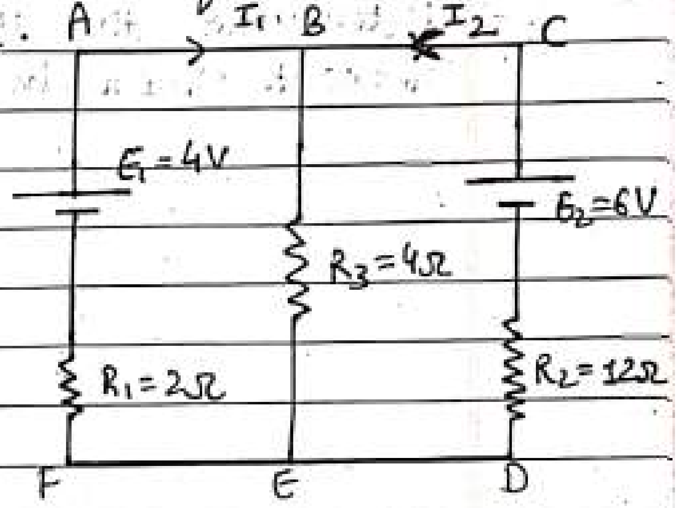
$\therefore$ $\boxed{I = 12A}$

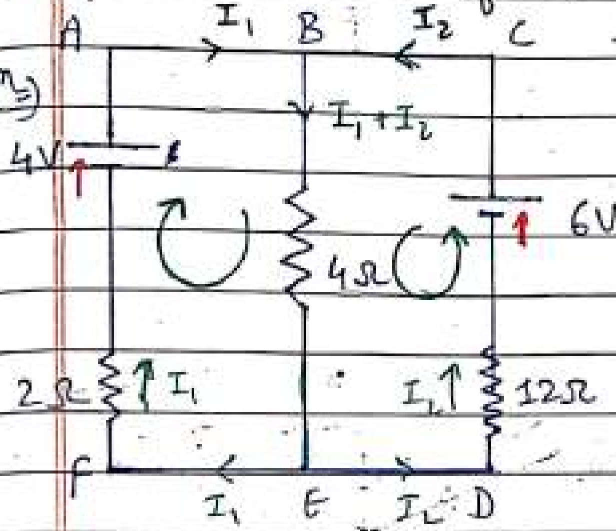
Current through AG, AF, AB $= \frac{I}{3} = 4A =$ current in CD, HD, ED

Current through GC, GE, BC, BH, FE, FH $= \frac{I}{6} = \frac{12}{6} = 2A$

Q ⇒ In the given circuit E_1 and E_2 are two cells of emf's 4V and 6V respectively, having negligible internal resistances. Applying Kirchoff's laws of electrical network, find the values of I_1 & I_2.



Sol^n ⇒



Applying Kirchoff's loop rule in loop ABEFA

$$\Sigma E = \Sigma IR$$
$$4 = 4(I_1 + I_2) + 2I_1$$
$$4 = 6I_1 + 4I_2$$
$$2 = 3I_1 + 2I_2 \quad \text{———} \; (1)$$

Applying Kirchoff's loop rule in loop CBEDC

$$6 = 4(I_1 + I_2) + 12I_2$$
$$6 = 4I_1 + 16I_2$$
$$3 = 2I_1 + 8I_2 \quad \text{———} \; (2)$$

Req^n (4 x eq^n (1)) − eq^n (2)

$$8 - 3 = (12I_1 + 8I_2) - (2I_1 + 8I_2)$$
$$5 = 10I_1 \Rightarrow \boxed{I_1 = \frac{1}{2}\,A}$$

Put $I = \frac{1}{2}$ in eq^n ①

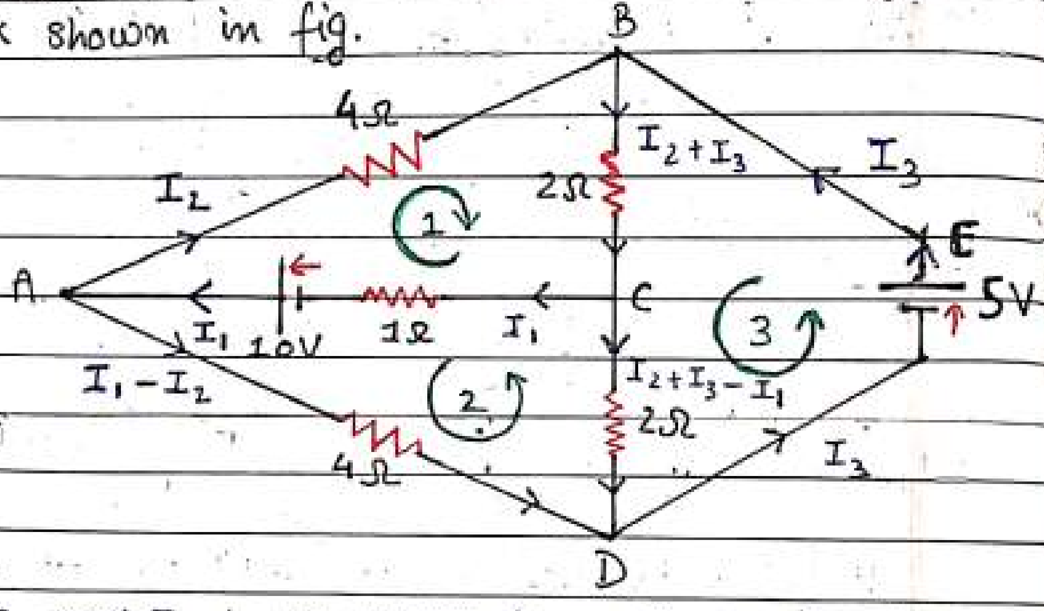
$$2 = \frac{3}{2} + 2I_2$$

$$2I_2 = 2 - \frac{3}{2} = \frac{1}{2}$$

$$\boxed{I_2 = \frac{1}{4}\ A}$$

NCERT

Q ⇒ Determine the current in each branch of the network shown in fig.



Sol^n ⇒ Let I_1, I_2 and I_3 be the currents as shown in fig.
Using Kirchoff's loop Rule
For Loop ①,

$$10 = 4I_2 + 2(I_2 + I_3) + I_1$$

$$10 = I_1 + 6I_2 + 2I_3 \quad \text{—— (i)}$$

For Loop ②, $$10 = I_1 + 4(I_1 - I_2) - 2(I_2 + I_3 - I_1)$$

$$10 = 7I_1 - 6I_2 - 2I_3 \quad \text{—— (ii)}$$

For Loop ③, $$5 = 2(I_2 + I_3) + 2(I_2 + I_3 - I_1)$$

$$5 = -2I_1 + 4I_2 + 4I_3 \quad \text{—— (iii)}$$

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eq^n (i) + eq^n (ii)

$$10 + 10 = (I_1 + 6I_2 + 2I_3) + (7I_1 - 6I_2 - 2I_3)$$

$$20 = 8I_1 \Rightarrow \boxed{I_1 = \frac{20}{8} = \frac{5}{2}\ A}$$

$\therefore$ eq^n (i) becomes, $10 = \frac{5}{2} + 6I_2 + 2I_3$

$$20 - 5 = 12I_2 + 4I_3$$

$$15 = 12I_2 + 4I_3 \text{ —— (iv)}$$

eq^n (iii) becomes, $5 = -5 + 4I_2 + 4I_3$

$$10 = 4I_2 + 4I_3 \text{ —— (v)}$$

eq^n (iv) − eq^n (v)

$$15 - 10 = (12I_2 + 4I_3) - (4I_2 + 4I_3)$$

$$5 = 8I_2 \Rightarrow \boxed{I_2 = \frac{5}{8}\ A}$$

Put $I_2 = \frac{5}{8}$ in eq^n (iv)

$$15 = \frac{15}{2} + 4I_3$$

$$8I_3 = 30 - 15 \Rightarrow \boxed{I_3 = \frac{15}{8}\ A}$$

The current in the various branches of network are:-

$I_{AB} = I_2 = \frac{5}{8}\ A = 0.625\ A$

$I_{CA} = I_1 = \frac{5}{2}\ A = 2.5\ A$

$I_{AD} = I_1 - I_2 = \frac{5}{2} - \frac{5}{8} = \frac{15}{8}\ A = 1.875\ A$

$I_{BC} = I_2 + I_3 = \frac{20}{8} = \frac{5}{2}\ A = 2.5\ A$

$I_{CD} = I_2 + I_3 - I_1 = \frac{5}{2} - \frac{5}{2} = 0\ A$

$I_{DGB} = I_3 = \frac{15}{8}\ A = 1.875\ A$

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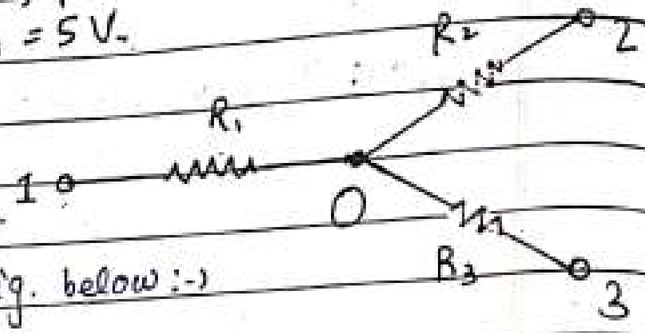
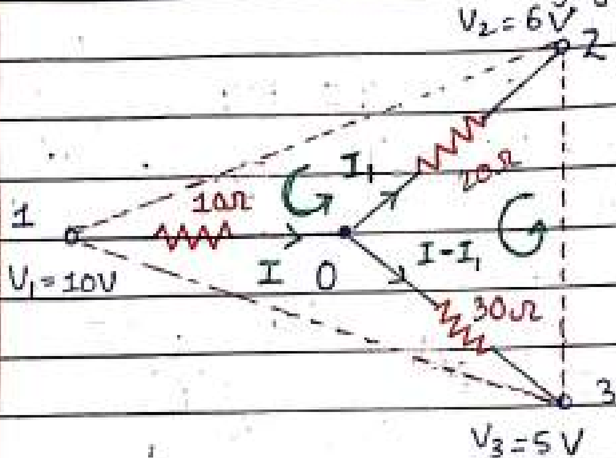
Q ⇒ Find the current flowing through the resistance R_1 of the circuit shown in fig. Given $R_1 = 10\Omega$, $R_2 = 20\Omega$ and $R_3 = 30\Omega$ and the potentials of points 1, 2 and 3 are $V_1 = 10V$, $V_2 = 6V$ and $V_3 = 5V$.



Sol^n ⇒ The distribution of current for the given network is shown in fig. below :-



Applying Kirchoff's Loop Rule to the loop 1021, we get

$$\Sigma E = \Sigma IR$$

$$10 - 6 = 10I + 20I_1$$

$$4 = 10I + 20I_1 \quad \text{——— (i)}$$

Applying Kirchoff's loop Rule to loop 0320.

$$6 - 5 = 30(I - I_1) - 20I_1$$

$$1 = 30I - 50I_1 \quad \text{——— (ii)}$$

[3 × eq^n(i)] − [eq^n(ii)], we get

$$12 - 1 = (30I + 60I_1) - (30I - 50I_1)$$

$$11 = 110I_1$$

$$I_1 = \frac{1}{10} = 0.1\,A$$

Putting $I_1 = \frac{1}{10}$ in eq^n (ii), we get

$$1 = 30I - \frac{50}{10} = 30I - 5$$

$$30I = 6$$

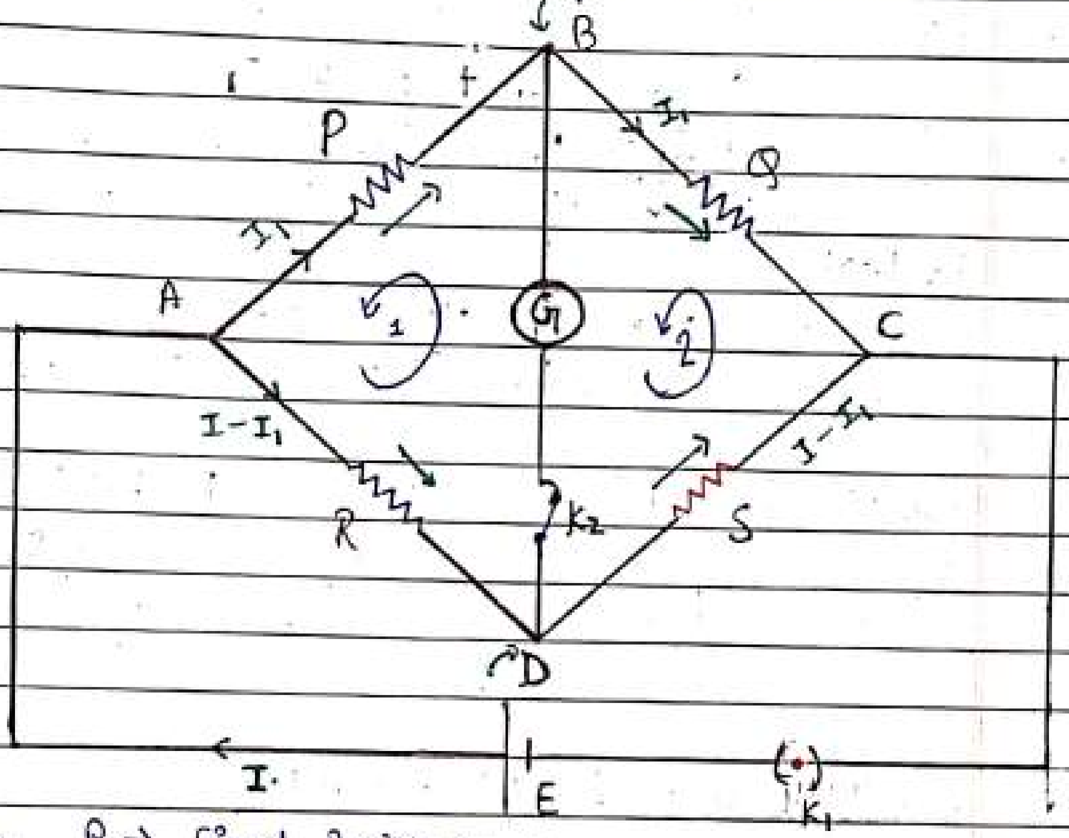
$$I = \frac{6}{30} = \frac{1}{5} \Rightarrow \boxed{I = 0.2\,A}$$

Thus current through R_1 is 0.2 A

WHEASTONE BRIDGE

'It is an arrangement of 4 resistances (3 known & 1 unknown) used to determine the value of unknown resistance in terms of the remaining 3 resistances.'

To do so the circuit is prepared as shown below:-



Here, P ⇒ Fixed Resistance
Q ⇒ Fixed "
R ⇒ Variable "
} Known Resistances.
S ⇒ Unknown "

A battery of emf E is connected b/w A and C and a sensitive galvanometer b/w B and D

Plug in the key K_1

The value of R is so adjusted that even on pressing the key K_2, no deflection is observed in the galvanometer ie, no current flows through the arm BD and hence points B and D are at the same potential.

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In this case, the Bridge is said to be balanced and in such condition,

$$\boxed{\frac{P}{Q} = \frac{R}{S}}$$

Proof :- In accordance with Kirchhoff's Law, the current through various branches is shown in fig.
Applying Kirchoff's Loop Rule

In Loop 1

$$0 = -PI_1 + R(I-I_1)$$
$$PI_1 = R(I-I_1) \quad \text{--- (1)}$$

In Loop 2

$$0 = -I_1Q + S(I-I_1)$$
$$QI_1 = S(I-I_1) \quad \text{--- (2)}$$

Eq.n (1) ÷ Eq.n (2) :

$$\frac{PI_1}{QI_1} = \frac{R(I-I_1)}{S(I-I_1)}$$

$$\boxed{\frac{P}{Q} = \frac{R}{S}}$$

Senstivity of a Wheatstone Bridge :-

'A wheatstone bridge is said to be sensitive if it shows a large deflection in the galvanometer for a small change of resistance in the resistance arm'.

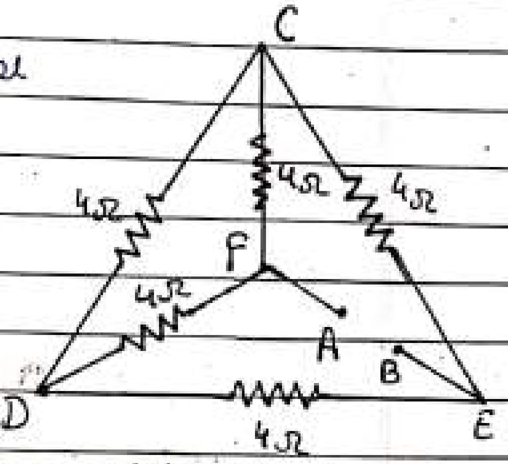
ADVANTAGES OF WHEATSTONE BRIDGE METHOD :-

(1) It is a null method. Hence the internal resistance of the cell and the resistance of the galvanometer do not affect the null point.

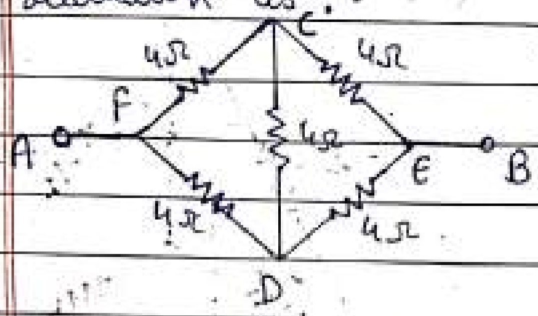
(2) The unknown resistance can be measured to a very high degree of accuracy by increasing the ratio of the resistances in arms P and Q.

(3) As the method does not involve any measurement of current and potential difference, so the resistance of ammeters and voltmeters do not affect the measurement.

Q ⇒ In the fig. shown below, all resistances are of 4Ω. Find the equivalent resistance between the point A and B. If a battery of e.m.f. 4V is connected across the points A & B, then find the currents through arms AFCEB & AFDEB.



Solⁿ ⇒ The given circuit network can be redrawn as :-

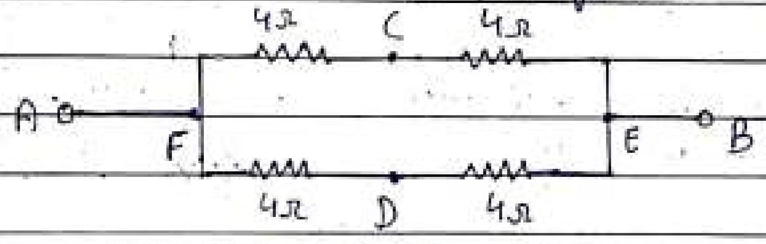


Clearly, it is a balanced wheatstone bridge as

$$\frac{4\Omega}{4\Omega} = \frac{4\Omega}{4\Omega}$$

Hence the points C & D are at same potential. The resistance in arm CD is ineffective.

So, the given network reduces to the equivalent circuit as shown

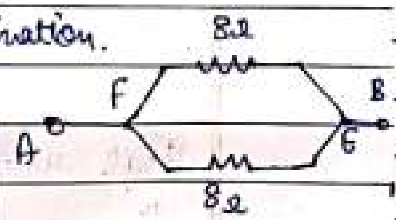


Total resistance along FCE $= 4 + 4 = 8\Omega$

" " " FDE $= 4 + 4 = 8\Omega$

These two resistances are in parallel combination.

∴ Equivalent resistance b/w A & B



$$\frac{1}{R_{eq}} = \frac{1}{8} + \frac{1}{8} \Rightarrow \boxed{R_{eq} = \frac{8}{2} = 4\Omega}$$

When a 4 V battery is connected across AB, then

Total current in the circuit $I = \frac{V}{R_{eq}}$

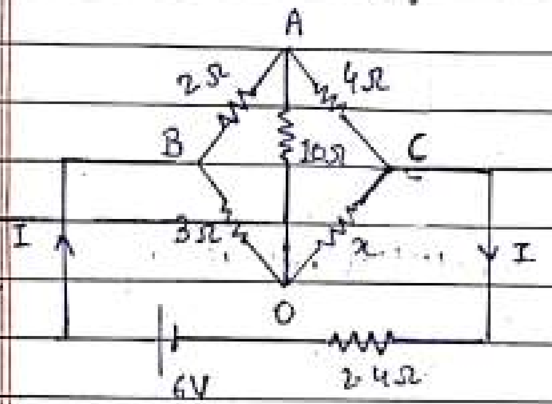
$$I = \frac{4}{4} = 1A$$

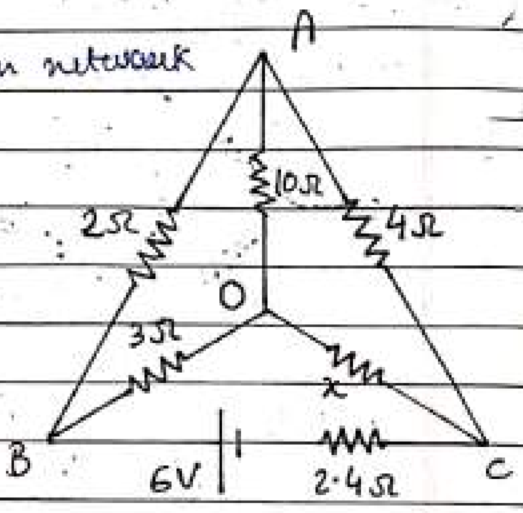
as, current in arm AFCEB = current in arm AFDEB

$\therefore$ " " " " $= \frac{1}{2}A = 0.5A$.

Q ⇒ Find the value of x if no current flows through the branch AO. Also find the current drawn from the battery.

Sol^n ⇒ The equivalent circuit for the given network is shown in the fig. below

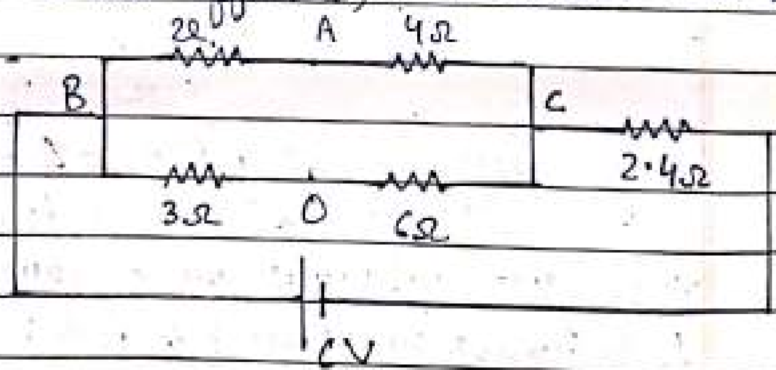




As no current flows through 10Ω resistance in branch AO, so the given circuit is a Balanced Wheatsone Bridge.

Hence, $\frac{2}{4} = \frac{3}{x}$ ⇒ $\boxed{x = \frac{3 \times 4}{2} = 6\Omega}$

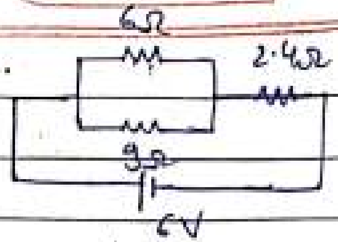
As 10Ω resistance is not effective, so the circuit can be redrawn as



Total resistance across BAC = $2 + 4 = 6\Omega$

" " " BOC = $3 + 6 = 9\Omega$

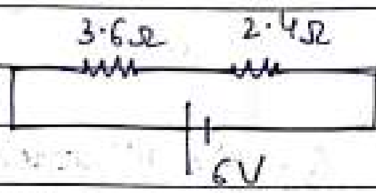
These two resistance are in ||el combination.
∴ Effective resistance b/w B & C is



$$\frac{1}{R} = \frac{1}{6} + \frac{1}{9} \Rightarrow R = \frac{6 \times 9}{6+9}$$

$$R = \frac{18}{5} = 3.6\Omega$$

3.6Ω & 2.4Ω are in series.

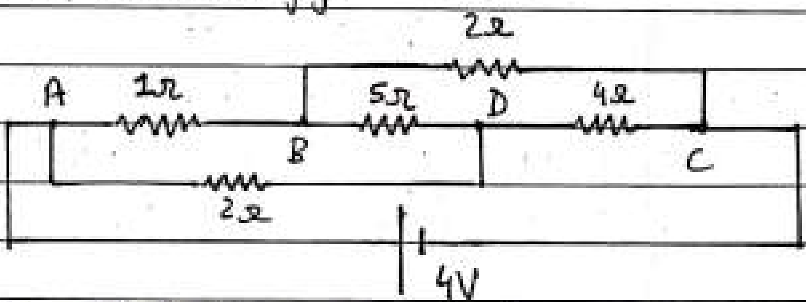


∴ Total Resistance in the circuit.

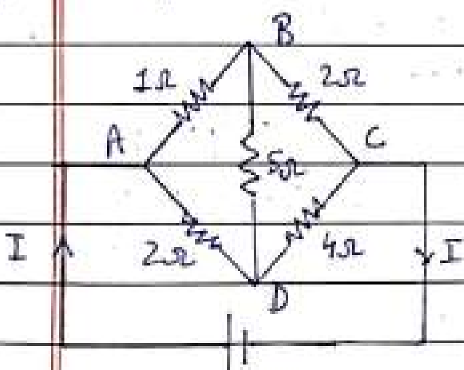
$$= 3.6 + 2.4 = 6\Omega$$

Hence, current in circuit $= \frac{V}{R} = \frac{6}{6} = 1A$.

Q⇒ Calculate the current drawn from the battery by the network of resistors as shown in fig.



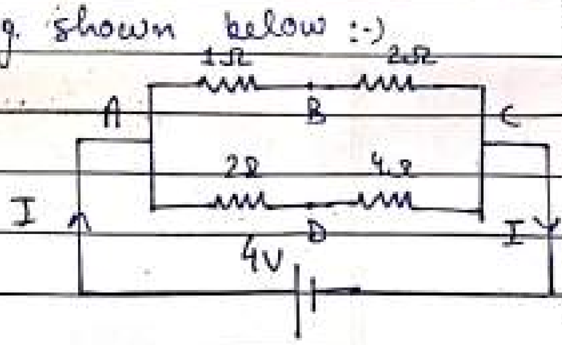
Sol^n⇒ The given network is equivalent to the circuit shown in fig.



Now, $\frac{1}{2} = \frac{2}{4}$ i.e. $\frac{P}{Q} = \frac{R}{S}$

Hence, the given circuit is a balanced wheatstone bridge. So, the resistance of 5Ω in arm BD is ineffective.

So, the equivalent circuit reduces to the circuit in fig. shown below :-)



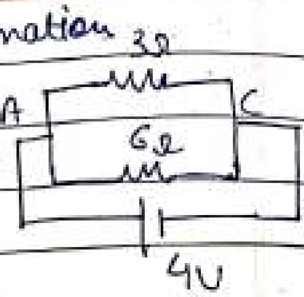
Net resistance in branch ABC.

$$= 1 + 2 = 3\Omega$$

Net resistance in branch ADC

$$= 2 + 4 = 6\Omega$$

Now, these 2 resistors are in ||el combination

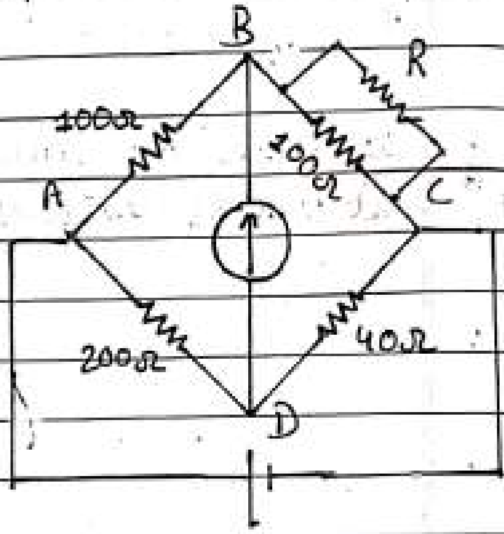


∴ equivalent resistance of the circuit is $\frac{1}{R_{eq}} = \frac{1}{3} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

$\boxed{R_{eq} = 2\,\Omega}$

Current in circuit $= \frac{\text{Total emf}}{\text{Total resistance}} = \frac{\cancel{8}\,4}{2} = 2A.$

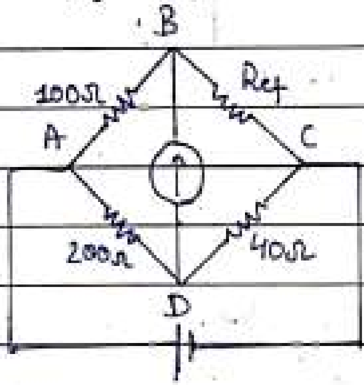
Q ⇒ The wheatstone's bridge in fig. is showing no deflection in the galvanometer joined b/w the points B & D. Compute the value of R.



Solⁿ ⇒ Effective resistance across the arm BC is

$R_{ef} = \frac{R \times 100}{R + 100} = \frac{100R}{100+R}$ — (1)

∴ The equivalent circuit reduces to the circuit shown in fig. below



As, the given circuit is a balanced wheatstone Bridge

$\therefore \frac{100\,\Omega}{R_{ef}} = \frac{200\,\Omega}{40\,\Omega}$

$\Rightarrow R_{ef} = \frac{100 \times 40}{200}\,\Omega$

$\Rightarrow \frac{100R}{100+R} = 20\,\Omega$

$\Rightarrow 100R = 2000 + 20R$

$\Rightarrow 80R = 2000$

$\Rightarrow R = \frac{2000}{80} = 25$

$\therefore \boxed{R = 25\,\Omega}$

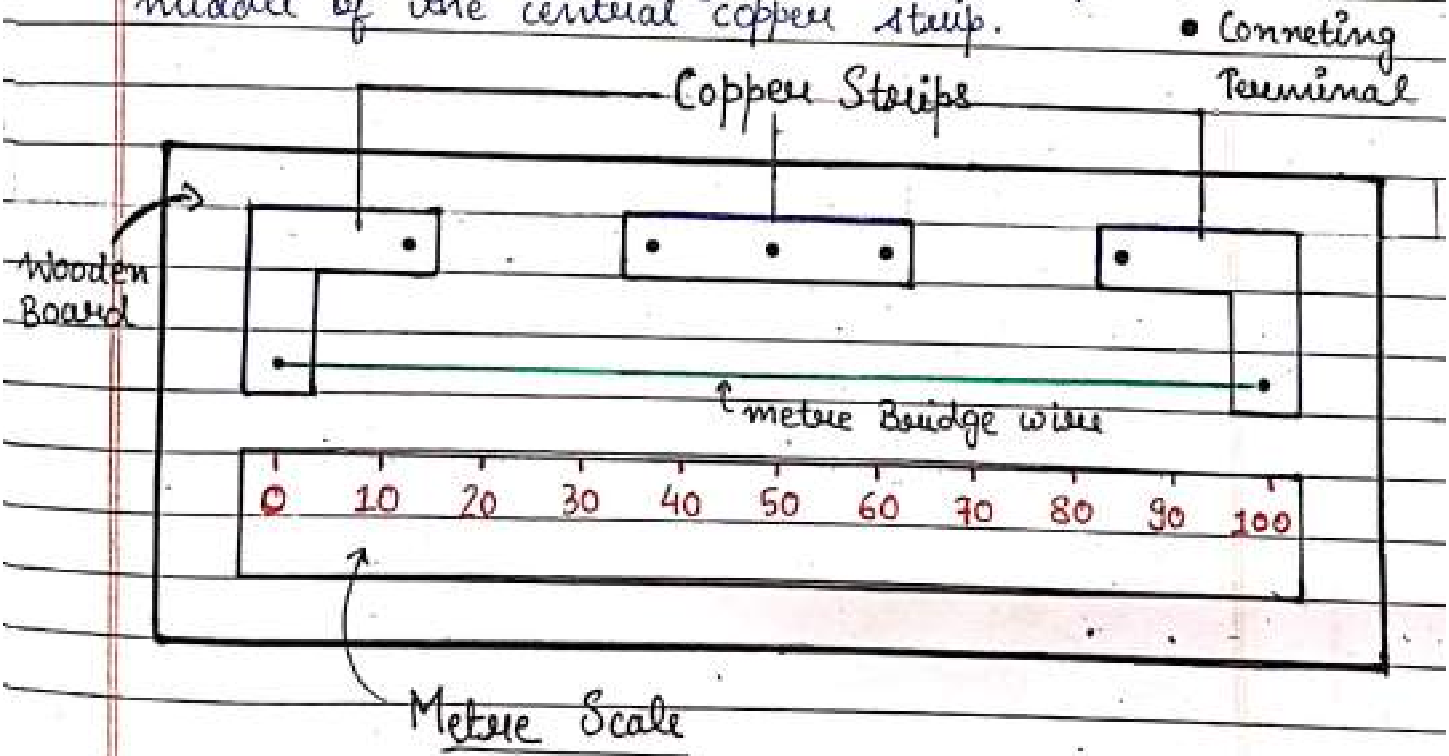
METRE BRIDGE / SLIDE WIRE BRIDGE

"It is a simple practical application of wheatstone bridge to measure unknown resistance."

⇒ **PRINCIPLE** :→ Its working is based on the principle of balanced wheatsone bridge; i.e.;

$$\frac{P}{Q} = \frac{R}{S}$$

⇒ **CONSTRUCTION** :→ It consists of usually 1 metre long constantan [or Manganin] wire of uniform cross-section, stretched between two L-shaped thick copper strips, along a metre scale fixed over wooden board. Another thick central copper strip is fitted on the board to provide 2 gaps, as shown in fig. Connecting screws are provided at the ends of the copper strips. A connecting screw is also provided at the middle of the central copper strip.



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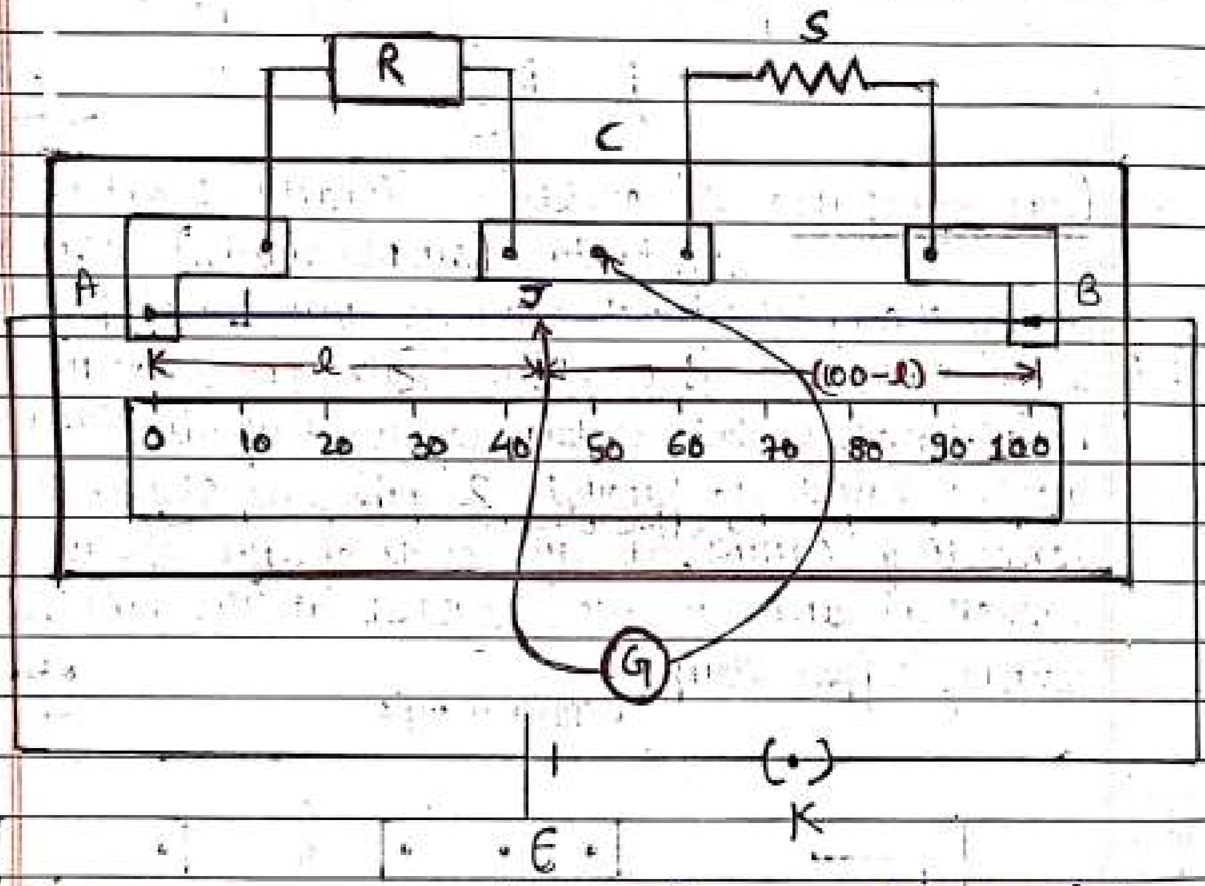
Applications of Metre Bridge

(a) To Measure an Unknown Resistance :→

To do so, a resistance box R and an unknown resistance S are connected in the 2 gaps. A source of emf E and a plug key are connected across the meter bridge wire AB. A movable jockey as J and a galvanometer G are connected as shown below.



Introduce some resistance in the left gap with the help of a resistance box R and adjust it in such a way that the balance point (at which when the jockey is touched, galvanometer gives zero deflection). Let J is the position of Balance point.

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When the bridge is balanced, let $AJ = l$.
then $JB = (100 - l)$.
In this case using the principle of Balanced Wheatstone Bridge,

$$\frac{\text{Resistance } R}{\text{Resistance } S} = \frac{\text{Resistance of the portion AJ of the wire}}{\text{Resistance of the portion JB of the wire}} \quad \text{--- (1)}$$

If 'x' is the resistance per unit length of the meter bridge wire then from eq^n (1)

$$\frac{R}{S} = \frac{lx}{(100-l)x} \quad \text{or} \quad \frac{R}{S} = \frac{l}{100-l}$$

$$\boxed{S = \frac{(100-l)\,R}{l}} \quad \text{--- (2)}$$

Knowing the values of R and l, unknown resistance S can be found.

(b) To Compare 2 unknown Resistance :→

Connect the unknown resistance R_1 in the place of S, in the fig shown and proceed in the same way. Let l_1 be the balancing length in this case. Then from eq^n (2)

$$R_1 = \frac{(100-l_1)\,R}{l_1} \quad \text{--- (3)}$$

Now, connect the unknown resistance R_2 in the place of R_1 and proceed in the same way. Let l_2 be the balancing length in this case. Then from eq^n (2)

$$R_2 = \frac{(100-l_2)\,R}{l_2} \quad \text{--- (4)}$$

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Dividing eq^n (3) and eq^n (4), we get

$$\boxed{\frac{R_1}{R_2} = \frac{l_2}{l_1}\frac{(100-l_1)}{(100-l_2)}} \longrightarrow (5)$$

(c) To measure the unknown Temperature :-

Refer to the circuit diagram in 1st application. Measure the values of resistance S at three different temperatures 0°C, 100°C and unknown temperature θ°C by immersing it in ice, steam and hot bath of unknown temperature respectively.
Let their respective values are S_0, S_{100} and S_θ.
As,

$$R = R_0(1+\alpha\theta) \longrightarrow (6)$$

$$\therefore \quad R - R_0 = R_0\alpha\theta \longrightarrow (7)$$

Hence $$S_{100} - S_0 = S_0\alpha\, 100 \longrightarrow (8)$$
and $$S_\theta - S_0 = S_0\alpha\theta \longrightarrow (9)$$

Dividing eq^n (9) by (8)

$$\frac{\theta}{100} = \frac{S_\theta - S_0}{S_{100} - S_0}$$

$$\boxed{\theta = \frac{S_\theta - S_0}{S_{100} - S_0} \times 100} \longrightarrow (10)$$

Substituting the values S_0, S_{100} and S_θ, unknown temperature can be determined.

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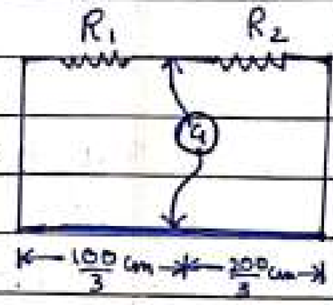
Q:) With the 2 resistances in the two gaps of metre bridge, the balance point is found to be at 100/3 cm from zero end. When 6 Ω resistance is connected in series with the smaller of the two resistances, the balance point is shifted to 200/3 cm from the same end. Find the resistance of the two wires.

Sol:) Here $l_1 = 100/3$ cm, $l_2 = 100 - \frac{100}{3} = \frac{200}{3}$ cm

According to case I :-

$$\frac{R_1}{R_2} = \frac{100/3}{200/3} \Rightarrow \frac{R_1}{R_2} = \frac{1}{2} \quad \text{——} (1)$$

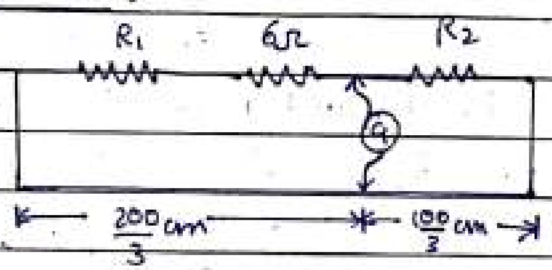


$$\Rightarrow 2R_1 = R_2$$

Clearly, $R_2 > R_1$

∴ 6Ω resistance will be connected with R_1

According to Case II :-



$$\frac{R_1+6}{R_2} = \frac{200/3}{100/3} \Rightarrow \frac{R_1+6}{R_2} = 2 \quad \text{——} (2)$$

from eq^n (1) and (2)

$$\frac{R_1+6}{2R_1} = 2$$

$$R_1 + 6 = 4R_1$$

$$3R_1 = 6$$

$$\boxed{R_1 = 2\Omega}$$

as $R_2 = 2R_1$

$$R_2 = 2 \times 2 \Rightarrow \boxed{R_2 = 4\,\Omega}$$

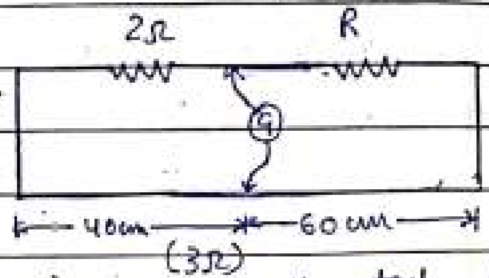
Q:) In a meter bridge when a resistance in the left gap is 2Ω and an unknown resistance in the right gap. The balance point is obtained at 40 cm from the zero end. On shunting the resistance with 2Ω, find the shift of the balance point on the wire.

Sol^n ⇒ Case I :→ $R_1 = 2\Omega$, $R_2 = R$, $l_1 = 40$ cm, $l_2 = 100-40$

$\therefore l_2 = 60$ cm

$$\therefore \frac{R_1}{R_2} = \frac{l_1}{l_2}$$

$$\frac{2}{R} = \frac{40}{60} \Rightarrow \boxed{R = 3\Omega}$$



Case II :→ when the unknown resistance is shunted with 2Ω.

$R_1 = 2\Omega$ $\quad R_2 = \frac{2\times 3}{2+3} = \frac{6}{5}\,\Omega$

Let 'l' be the balancing length from the zero end.

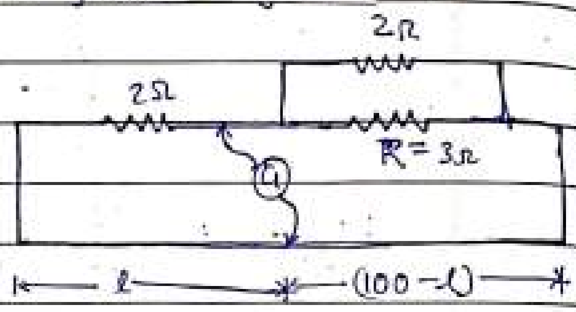
$$\therefore \frac{2}{\frac{6}{5}} = \frac{l}{100-l}$$

$$\cancel{10}\ \frac{5}{3} = \frac{l}{100-l}$$

$$500 - 5l = 3l$$

$$8l = 500$$

$$l = \frac{500}{8} = 62.5 \text{ cm}$$

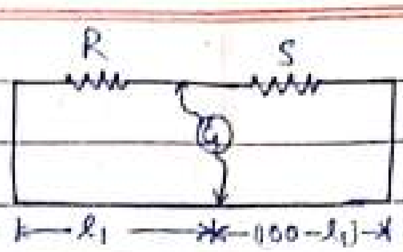


$\therefore$ Shift in the balance point $= 62.5 - 40$

$= 22.5$ cm Ans.

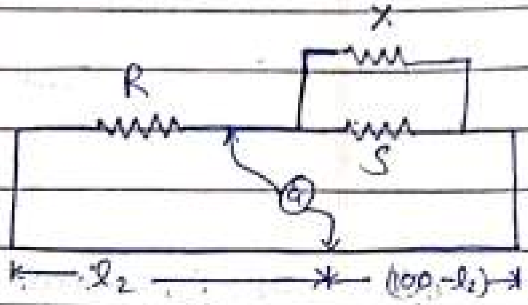
Q ⇒ When two known resistances R and S are connected in the left and the right gaps of a meter bridge, the balance point is found at a distance l_1 from the zero end of the meter bridge wire. An unknown resistance X is now connected in parallel to the resistance ~~R~~ S, and the balance point is now found at a distance l_2 from the same end. Obtain the formula for X in terms of l_1, l_2 and S.

Sol^n :-> Case I :-> when only R & S are connected.



$$\frac{R}{S} = \frac{l_1}{100-l_1} \quad \text{—— } (1)$$

Case II :-> when X is connected in parallel with S.



$$\frac{R}{\frac{1}{\frac{1}{X}+\frac{1}{S}}} = \frac{l_2}{100-l_2}$$

$$\frac{R}{\frac{XS}{X+S}} = \frac{l_2}{100-l_2}$$

$$\frac{R(X+S)}{SX} = \frac{l_2}{100-l_2} \quad \text{—— } (2)$$

from eq^n (1) $\frac{R}{S} = \frac{l_1}{100-l_1}$

∴ eq^n (2) becomes

$$\frac{l_1}{(100-l_1)}\frac{(X+S)}{X} = \frac{l_2}{100-l_2}$$

$$\frac{l_1X}{X} + \frac{l_1S}{X} = \frac{l_2}{100-l_2}(100-l_1)$$

$$l_1 + \frac{l_1S}{X} = \frac{l_2}{100-l_2}(100-l_1)$$

$$\frac{l_1S}{X} = \frac{l_2(100-l_1)}{100-l_2} - l_1$$

$$\frac{l_1S}{X} = \frac{100l_2 - \cancel{l_1l_2} - 100l_1 + \cancel{l_1l_2}}{(100-l_2)}$$

$$\frac{l_1S}{X} = \frac{100(l_2-l_1)}{100-l_2}$$

$$\boxed{X = \frac{Sl_1(100-l_2)}{100(l_2-l_1)}}$$

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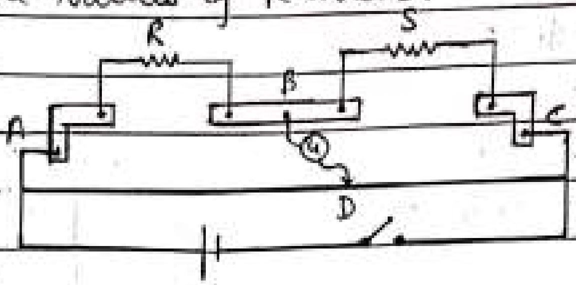
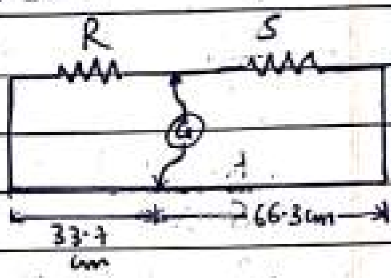
Q ⇒ In a metre bridge (shown in fig.), the null point is found at 33.7 cm from A. If now a resistance of 12 Ω is connected in parallel with S, the null point occurs at 51.9 cm. Determine the values of R and S.



Solⁿ ⇒ Case I :→ when no shunt is connected

$$\frac{R}{S} = \frac{33.7}{66.3} \quad \text{——} \; ①$$



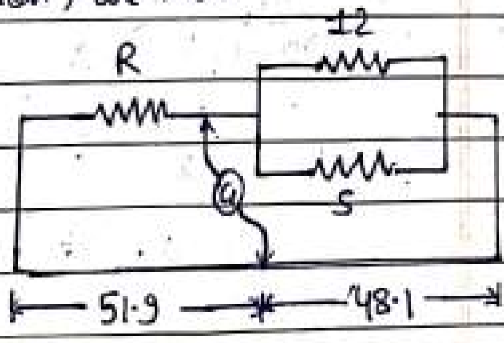
Case II :→ when S is shunted with a resistance of 12 Ω.

$\therefore$ Net resistance of S = $S_{eq} = \dfrac{12S}{S+12}$

with the new balance condition, we have

$$\frac{R}{S_{eq}} = \frac{51.9}{48.1}$$

$$\frac{R(S+12)}{12S} = \frac{51.9}{48.1} \quad \text{——} \; ②$$



~~S+12~~

Dividing ① & ②

$$\frac{12}{S+12} = \frac{33.7}{66.3} \times \frac{48.1}{51.9}$$

$$\frac{12}{S+12} = 0.471$$

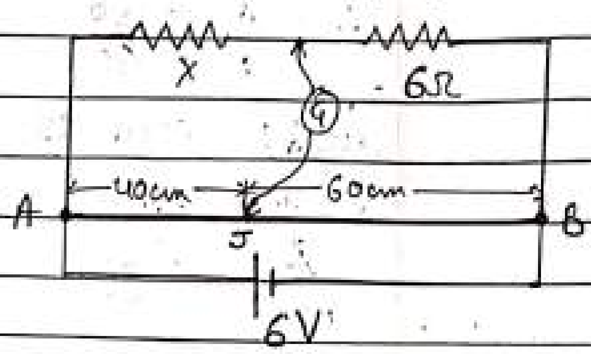
$$\frac{12}{0.471} = S + 12 \quad \Rightarrow \quad S + 12 = 25.47$$

$$\boxed{S \approx 13.47\,\Omega}$$

$$\therefore \quad R = \frac{33.7}{66.3} \times 13.47$$

$$\Rightarrow \boxed{R \approx 6.84\,\Omega}$$

Q:) In the following circuit, a metre bridge is shown in its balanced state. The metre bridge wire has a resistance of **1 Ω cm**. Calculate the value of the unknown resistance X and the current drawn from the battery of negligible internal resistance.



Sol^n: In balanced condition

$$\frac{X}{6} = \frac{40}{60}$$

$$X = \frac{2}{3} \times 6$$

$$\boxed{X = 4\,\Omega}$$

Resistance of 1cm wire = 1Ω

" " 100cm wire (AB) = 100 Ω

Total resistance of resistances X and 6 Ω connected in series = 4+6 = 10Ω

This series combination is in ||el with wire AB

∴ Equivalent Resistance $= \frac{10 \times 100}{10+100} = \frac{100}{11}\,\Omega$

emf of the battery = 6 V

∴ Current drawn from the battery $= \frac{\text{total emf}}{\text{total resistance}}$

$$I = \frac{6}{\frac{100}{11}} = 0.66A$$

Q:) In a metre bridge, the null point is found to be 60 cm away from the zero end with resistance X in left gap and Y in right gap. When a resistance of 15Ω is connected in series with Y, the null point is found to shift by 10 cm towards

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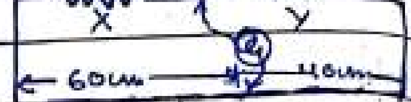
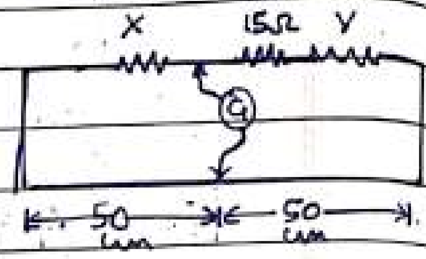
the zero end of the wire. Find the position of null point if a resistance of 30Ω were connected in parallel with Y.

Solⁿ → Case I :→ when no resistance is connected with X & Y

$$\therefore \frac{X}{Y} = \frac{60}{40} \quad \text{or} \quad \frac{X}{Y} = \frac{3}{2}$$



Case II :→ when 15Ω resistance is connected with Y in series



In this case,

$$\frac{X}{Y+15} = \frac{50}{50} = 1$$

$$\frac{Y+15}{X} = 1$$

$$\frac{Y}{X} + \frac{15}{X} = 1 \Rightarrow \frac{2}{3} + \frac{15}{X} = 1 \quad \left(\because \frac{X}{Y} = \frac{3}{2}\right)$$

$$2X + 45 = 3X$$

$$\boxed{X = 45\Omega}$$

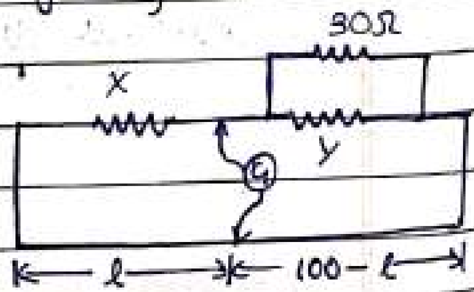
$$\therefore Y = \frac{45 \times 2}{3} = 30 \Rightarrow \boxed{Y = 30\Omega}$$

when a resistance of 30Ω is connected in ||el with Y, the resistance in the right gap becomes

$$Y' = \frac{30Y}{30+Y} = \frac{30 \times 30}{30+30} = 15\Omega$$

Suppose 'l' is the balancing length from zero end

Then, $\frac{X}{15} = \frac{l}{100-l}$



$$\frac{45}{15} = \frac{l}{100-l}$$

$$300 - 3l = l$$

$$4l = 300$$

$$\boxed{l = 75 cm}$$

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POTENTIOMETER

'It is a device used to measure an unknown emf or potential difference accurately.'

Its other applications are :→

(1) To compare the emf of 2 cells.

(2) To measure the internal resistance of a cell.

PRINCIPLE :→ The working of potentiometer is based on the principle that 'when a constant current is passed through a wire of uniform area of cross section, the potential difference across any portion of the wire is directly proportional to the length of that portion.'

Let 'V' is the potential difference across a portion of the wire of resistance 'R'. If 'I' is the current through the wire, then

$$V = IR \quad \text{——— (1)}$$

If l, A and ρ be the length, area of cross section and resistivity of the material of the wire, then

$$R = \rho \frac{l}{A} \quad \text{——— (2)}$$

From eq^n (1) and (2)

$$V = I \rho \frac{l}{A}$$

If I and A are constant and ρ is also a constant, then,

$$V = \text{constant} \times l$$

or $V = kl$ [where $k = \frac{V}{l}$ is called potential gradient of wire]

$$\therefore \boxed{V \propto l}$$

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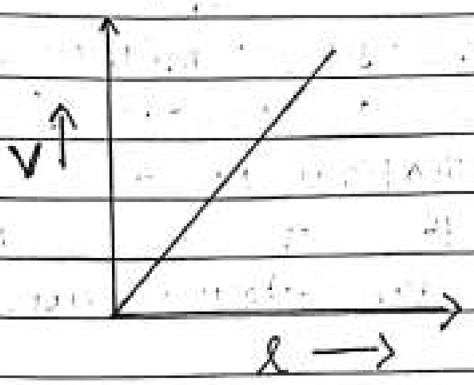
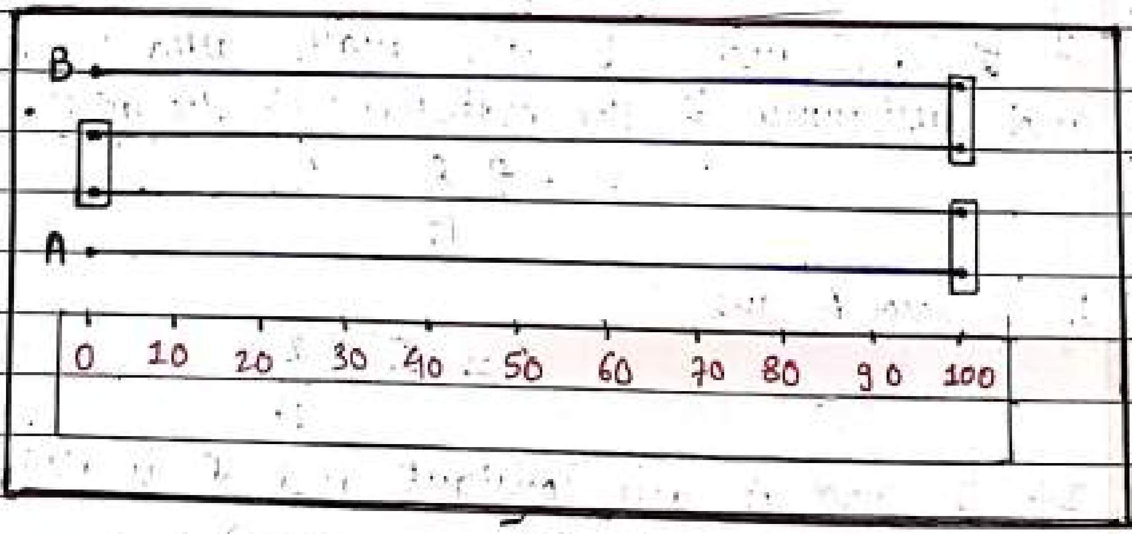
Potential Gradient (K)	⇒ Potential drop per unit length of the potentiometer wire.



CONSTRUCTION :→ It consists of a long wire AB of uniform cross section usually 4 to 10 m long of material having **high resistivity** and low **temperature coefficient** such as constantan or Manganin. Usually 1 m long pieces of wire are joined in-series through thick copper strips and fixed on a wooden board. A meter scale is fixed parallel to the wires. 2 connecting screws are provided at the ends A and B of the wire.



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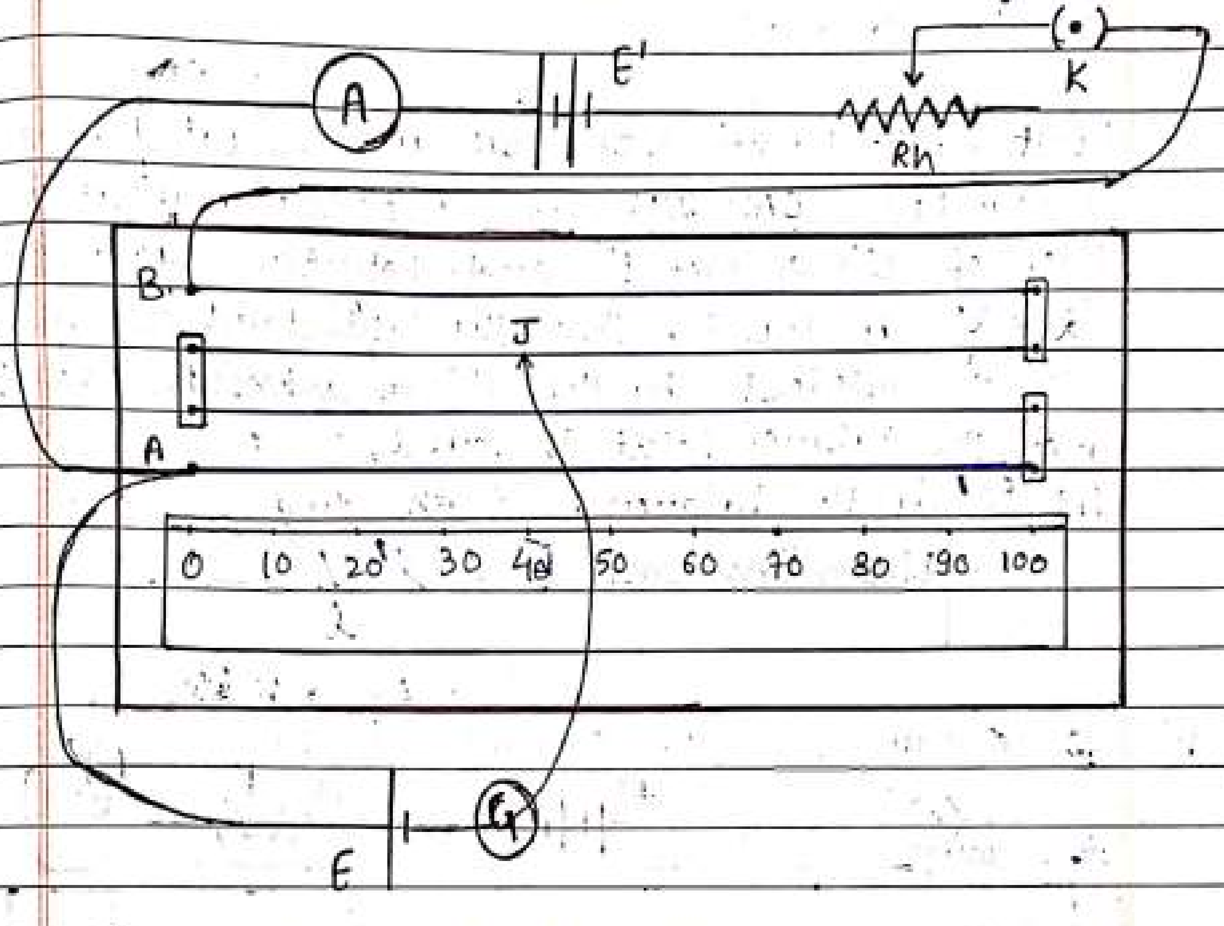
Applications of Potentiometre :→

*(a) To measure the e.m.f of cell :→

For this the ends A and B are connected to a strong battery, a plug key K. and a rheostat Rh. Here E is the unknown and E' is the known emf [E'>E].



After making the circuit, find the balance point on the potentiometer wire using jockey and the galvanometer. Let J be the position of balance position of the balance point. and let 'l' be the balancing length. If 'I' is the constant current in the potentiometer wire, then

Unknown emf E = potential drop across the balancing length of the potentiometer wire.

$E = IR_{AJ}$ [where R_{AJ} is the resistance of the portion AJ of the wire]

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If x is the resistance per unit length of the wire, then $R_{AJ} = lx$,

$\therefore$ $\boxed{E = Ilx}$ —— (1)

Knowing the values I, l and x the value of unknown emf can be found.

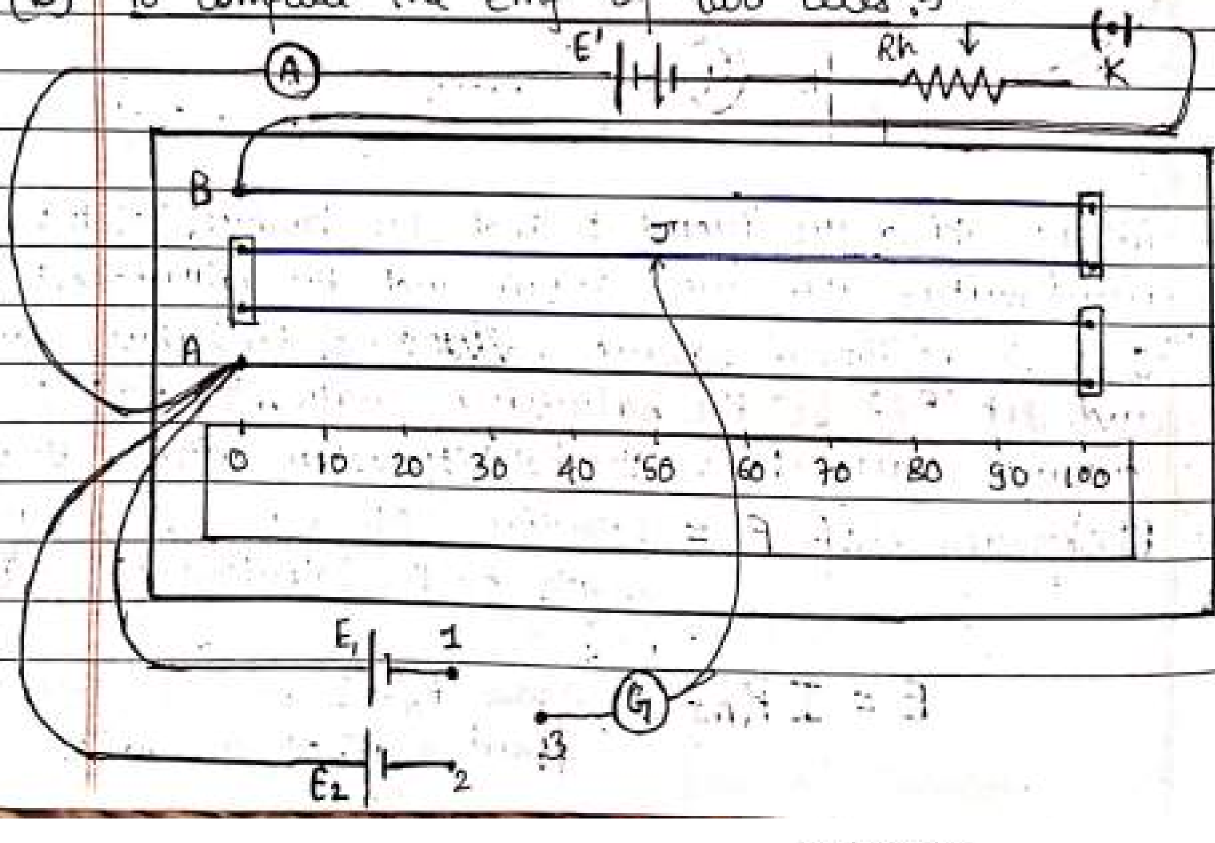
Another Method :-

Firstly the potentiometer wire is calibrated by using a standard CADMIUM cell (by connecting it at the place of cell of emf E) and potential gradient $\left(K = \frac{V}{l}\right)$ is found. Then the standard cell is replaced by the cell of unknown emf E and the balance point is found.

Let l' is the balancing length, then

$$\boxed{\text{Unknown emf } E = l' \frac{V}{l}} \quad \text{—— (2)}$$

(b) To Compare the emf of two cells :-



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For comparing the emf's of the two cells the circuit is prepared as shown.

Let E_1 and E_2 are the emfs to be compared and E' be the emf of the driver cell. A constant current 'I' is maintained in the potentiometer wire.

When the plug is put in the gap b/w the terminals 1 and 3 of the two way key, the cell of emf E_1 will come in the circuit. Let 'l_1' be the balancing length in this case, then

$$E_1 = I l_1 x \quad \text{———} \quad (1)$$

where x is the resistance per unit length of the potentiometer wire.

When the plug is put b/w the gap of terminals 2 and 3 of the two way key, the cell of emf E_2 will come in the circuit. Let 'l_2' be the balancing length in this case, then

$$E_2 = I l_2 x \quad \text{———} \quad (2)$$

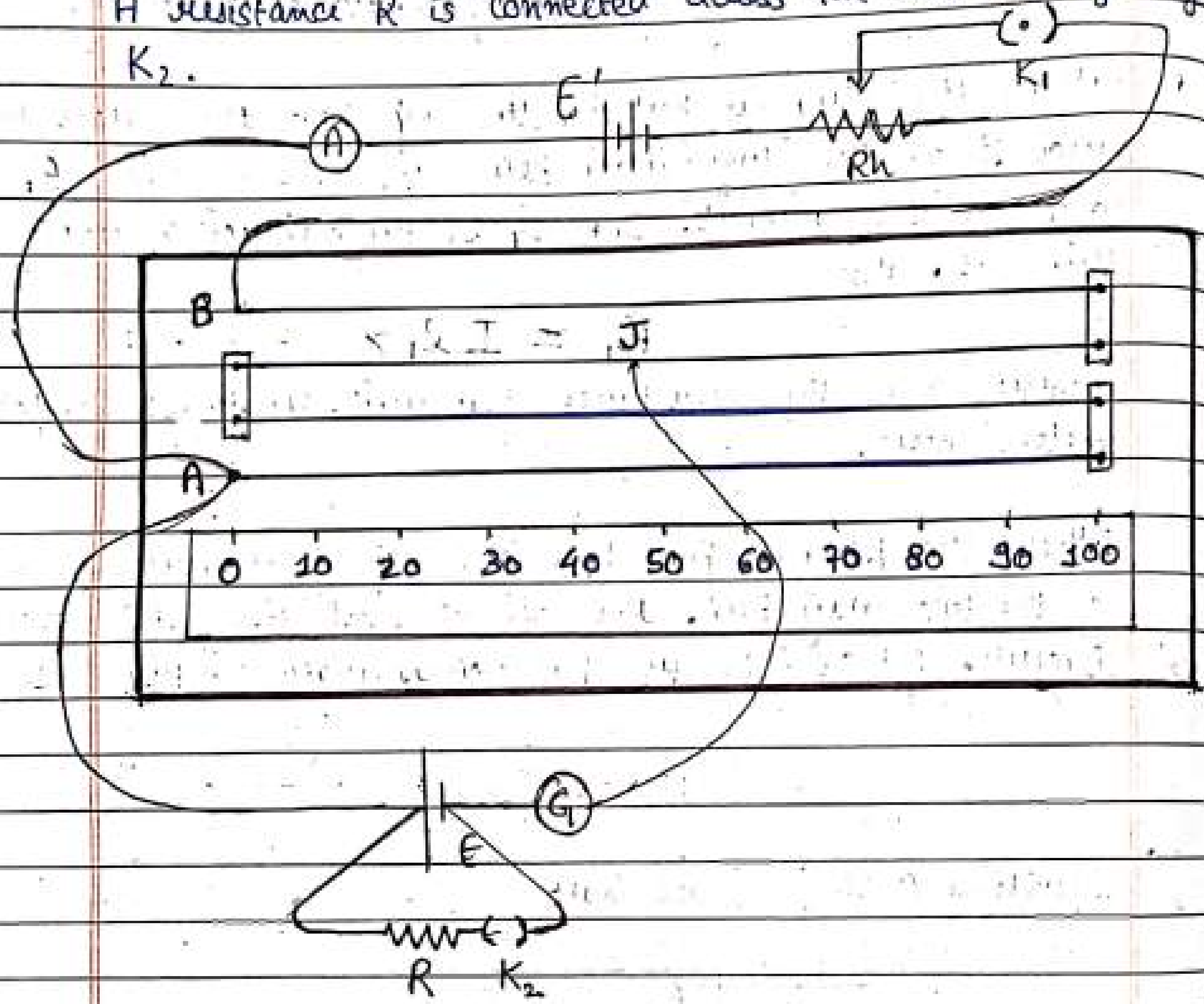
Dividing (1) & (2), we have

$$\boxed{\frac{E_1}{E_2} = \frac{l_1}{l_2}} \longleftarrow (3)$$

After measuring the values of l_1 and l_2, the ratio of the emfs of two cells can be found.

(C) To measure the internal resistance of the cell :-

To measure the internal resistance of the cell of emf E, the circuit is prepared as shown in fig. A resistance R is connected across the cell through key K_2.



Maintain a constant current I in the potentiometer wire and find the balancing length keeping the key K_2 <u>OPEN</u>.

Let 'l_1' be the balancing length in this case. As no current is drawn from the cell of emf E, therefore the emf E is balanced in this case, hence

$$E = I l_1 x \quad \text{—— (1)}$$

where x is the resistance per unit length of the potentiometer wire.

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Plug in the key K_2 and find the balancing length again. Let 'l_2' be the balancing length in this case. As the current is drawn from the cell of emf E, therefore in this case the TERMINAL POTENTIAL DIFFERENCE is balanced, hence

$$V = I l_2 x \quad \text{——} \quad (2)$$

Dividing eq^n (1) & (2), we get

$$\frac{E}{V} = \frac{l_1}{l_2} \quad \text{——} \quad (3)$$

As the internal resistance 'r' of the cell is given by

$$r = \left[\frac{E}{V} - 1\right] R \quad \text{——} \quad (4)$$

$\therefore$ from (3) & (4)

$$\boxed{r = \left[\frac{l_1}{l_2} - 1\right] R} \quad \text{——} \quad (5)$$

SENSTIVITY OF POTENTIOMETRE:-

A potentiometer is sensitive if :-

(i) It is capable of measuring small Potential Difference.

(ii) It shows a significant change in balancing length for a small change in potential difference being measured.

'Smaller the potential gradient, greater will be the senstivity of the potentiometer.'

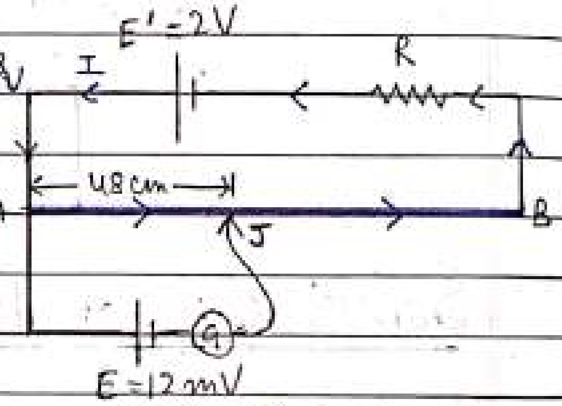
To increase the sensitivity :-

(i) For constant V, $k \propto \frac{1}{l}$ $\therefore$ $l\uparrow$ & $k\downarrow$

$\therefore$ On increasing the length of Potentiometer wire, its sensitivity increases.

(ii) For fixed length (l), K can be reduced by reducing the current in the circuit with the help of rheostat.

Q ⇒ A potentiometer wire of length 100 cm has resistance of 10 Ω. It is connected in series with a resistance and a cell of e.m.f. 2V and of negligible internal resistance. A source of emf of 12 mV is balanced against a length of 48 cm of the potentiometer wire. Find the value of the external resistance.



Sol^n ⇒ $l_{AB} = 100\,cm = 1\,m$

$R_{AB} = 10\,\Omega$ $\quad E = 12\,mV = 12\times10^{-3}\,V$

$\therefore$ Resistance per unit length

$x = \frac{10}{1} = 10\,\Omega/m$

Current in the potentiometer wire $I = \frac{2}{R+R_{AB}} = \frac{2}{R+10}$

$$I = \frac{2}{R+10} \quad \text{—— (1)}$$

as emf of cell = Potential difference across balancing length

$$12\times10^{-3} = I \times R_{AJ}$$

$$12\times10^{-3} = \frac{2}{R+10} \times l_{AJ} \times x$$

$$12\times10^{-3} = \frac{2}{R+10}\times\frac{48}{10\cancel{0}}\times 1\cancel{0}$$

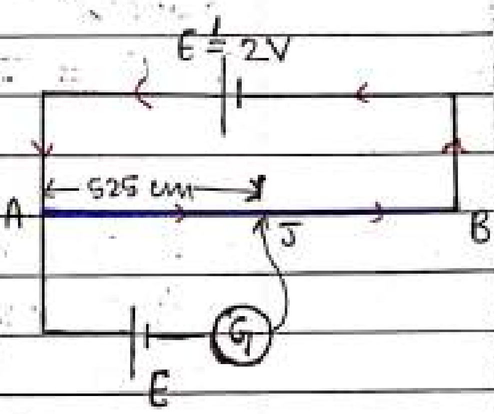
$$4\ \frac{\cancel{12}}{\cancel{48}}\times\frac{10^{-3}}{2}\times 10 = \frac{1}{R+10}$$

$$\frac{8}{10^{-2}} = R+10$$

$$R+10 = 800 \Rightarrow \boxed{R = 790\Omega}$$

Q=> A potentiometer wire 1000 cm long has a resistance of 20Ω. An accumulator of emf 2V and negligible internal resistance is connected across it. What is the potential drop per cm of the potentiometer wire and what is the emf of the cell which balances against a length of 525 cm?

Solⁿ:- $l_{AB} = 1000$ cm $\quad R_{AB} = 20\Omega$



Resistance per unit length $(x) = \frac{R_{AB}}{l_{AB}}$

$$x = \frac{20}{1000} = 0.02\Omega/cm$$

as potential of 2V is lost across a length of 1000 cm

(i) $\therefore$ Potential drop per cm $= \frac{2}{1000} = 2\times10^{-3}$ V/cm

(ii) current in the potentiometer wire $I = \frac{2}{20} = \frac{1}{10}$ A

Now, emf E = P.D across balancing length

$$E = I\times R_{AJ}$$

$$E = \frac{1}{10}\times[l\times x]$$

$$E = \frac{1}{10}\times 525\times 0.02$$

$$E = 52.5\times 0.62$$

$$\boxed{E = 1.05V}$$

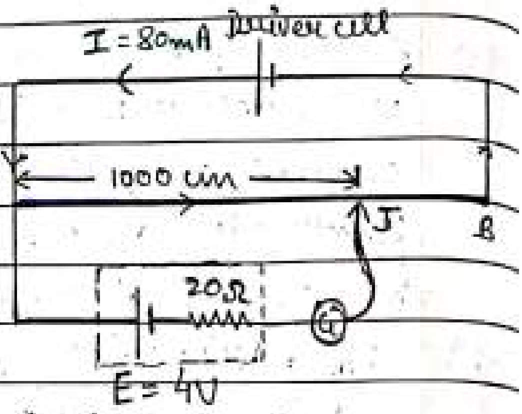
Q:- The length of a potentiometer wire is 1200cm and it carries a current of 80 mA. For a cell of e.m.f 4V and internal resistance 20Ω, the null point is found to be at 1000 cm. If a voltmeter is connected across the cell, the balancing length decreases by 20cm. Find :-

(a) the resistance of the potentiometer wire

(b) reading of the voltmeter

(c) resistance of the voltmeter.

$I = 80mA = 80 \times 10^{-3} A$



(a) Case I :- When voltmeter is not connected.

In this case emf E is balanced

$\therefore \quad E = I \times R_{AJ}$

R_{AJ} → Resistance of balancing length of wire.

$E = I \times l_{AJ} \times x$ ... [x → resistance per unit length of potentiometer wire]

$4 = 80 \times 10^{-3} \times 1000 \times x$

$x = \dfrac{4}{8 \times 10^{-2} \times 10^{3}}$

$x = \dfrac{1}{2} \times \dfrac{1}{10} = \dfrac{1}{20}\ \Omega/cm$

Length of potentiometer wire $l_{AB} = 1200\ cm$

$\therefore$ Resistance of the wire of length AB $= x \times l_{AB}$

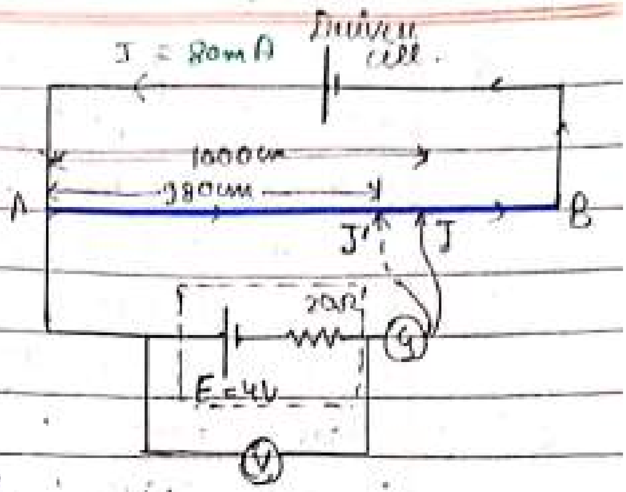
$R_{AB} = \dfrac{1}{20} \times 1200$

$\boxed{R_{AB} = 60\Omega}$

(b) Case II :- When voltmeter is connected across the cell of emf 4V.

In this case terminal potential difference of cell V is balanced

As balancing length decreates by 20 cm.



$\therefore l_{AJ'} = l_{AJ} - 20$

$l_{AJ'} = 1000 - 20$

$l_{AJ'} = 980\,cm$

$\therefore V = I\, l_{AJ'} \times x$

$V = 80 \times 10^{-3} \times 98\cancel{0} \times \frac{1}{2\cancel{0}}$

$V = \frac{\overset{4}{\cancel{8}} \times 98 \times 10^{-2}}{\cancel{2}} \Rightarrow \boxed{V = 3.92 \text{ Volts.}}$

(c) as $r = \left[\frac{E}{V} - 1\right] R$

$\therefore r = \left[\frac{E}{V} - 1\right] R_V$ [$R_V \rightarrow$ Resistance of Voltmeter]

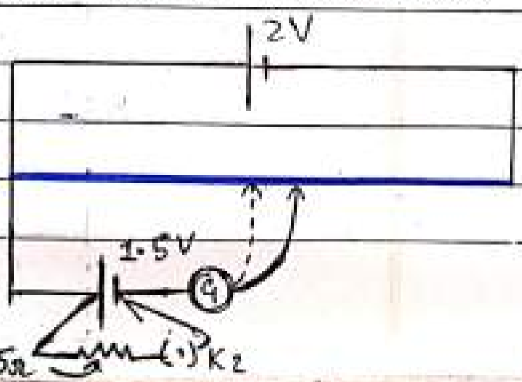
$20 = \left[\frac{4}{3.92} - 1\right] R_V$

$20 = \left[\frac{4 - 3.92}{3.92}\right] R_V$

$R_V = \frac{20 \times 3.92}{0.08} = \frac{20 \times \overset{49}{\cancel{392}}}{\cancel{8}} = 20 \times 49$

$\boxed{R_V = 980\,\Omega}$

Q⇒ In the fig. shown, the balancing length in open circuit is 76.3 cm. When the circuit is closed by closing the key K_2, the balancing length becomes 64.8cm. What is the internal resistance of the cell.



Sol^n: $l_1 = 76.3\,cm$ $l_2 = 64.8\,cm$

as $r = \left[\frac{l_1}{l_2} - 1\right] R$

$r = \left[\frac{76.3}{64.8} - 1\right] \times 9.5$

$\boxed{r = 1.69\,\Omega}$

Q⇒ A 10m long potentiometer wire has a potential gradient of 0.0025V/cm along its length. Calculate the length of wire at which null point is obtained for a 1.025V cell. Also find the emf of another cell for which the null point is obtained at 860cm. What maximum potential difference can be measured by this potentiometer.

Solⁿ (i) for 0.0025 V , length = 1 cm

$\therefore$ for 1.025 V , length $= \frac{1}{0.0025} \times 1.025$

$= 410\text{ cm} = 4.1\text{ m}$

(ii) for 1cm , emf = 0.0025

for 860cm, emf = 0.0025×860

$= 2.15\text{V}$

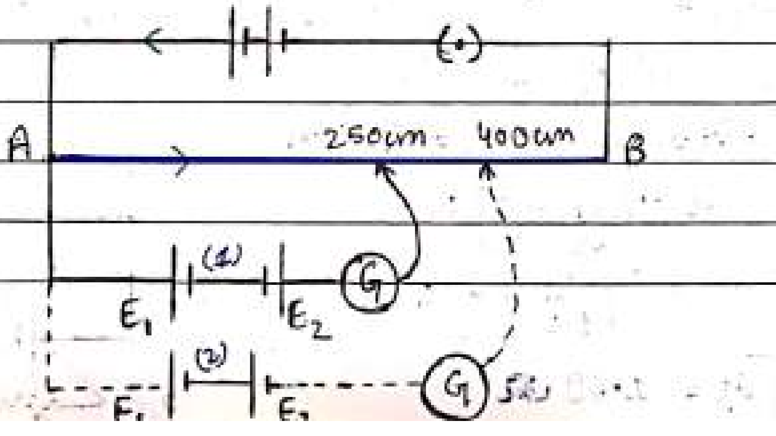
(iii) Maximum potential difference is measured at $l = 10\text{m}$ = length of potentiometer wire.

i.e. $l = 10 \times 100 = 1000\text{ cm}$

$\therefore E_{max} = 0.0025 \times 1000$

$E_{max} = 2.5\text{V}$

Q⇒ Two primary cells of emf E_1 and E_2 ($E_1 > E_2$) are connected to the potentiometer wire AB, as shown in the fig. If the balancing lengths for the two combinations of the cells are 250 cm and 400 cm, find the ratios of E_1 and E_2.



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Sol^n $l_1 = 250cm$ $\quad$ $l_2 = 400cm$

Let 'I' be the current through the circuit and 'x' be the resistance per unit length of the wire

$\therefore$ For 1st combination

$$E_1 - E_2 = I l_1 x$$
$$E_1 - E_2 = 250 I x \quad \text{——} \quad (1)$$

For 2nd combination

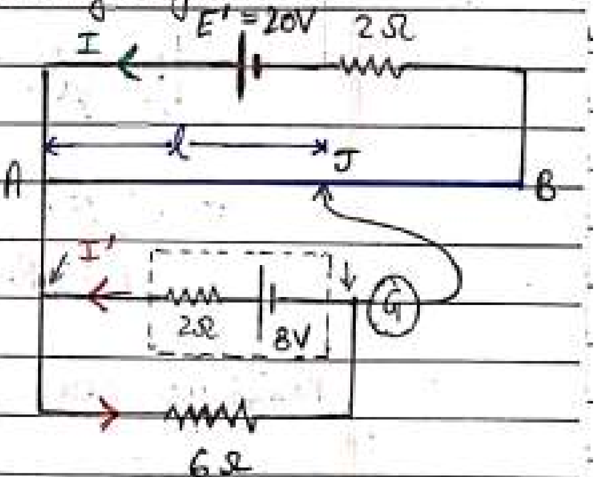
$$E_1 + E_2 = I l_2 x$$
$$E_1 + E_2 = 400 I x \quad \text{——} \quad (2)$$

Dividing eq^n (1) by (2).

$$\frac{E_1 - E_2}{E_1 + E_2} = \frac{250}{400} = \frac{5}{8}$$

$$8E_1 - 8E_2 = 5E_1 + 5E_2$$
$$3E_1 = 13E_2$$
$$\boxed{\frac{E_1}{E_2} = \frac{13}{3}} \text{ — Ans.}$$

Q=) In the fig. shown below, the wire AB has a length of 100cm. and resistance 8 Ω. Find the balancing length 'l'.



Sol^n=) Current in the potentiometer wire $I = \frac{E'}{2 + R_{AB}}$

$$I = \frac{20}{8+2} = 2A$$

current drawn by 6Ω resistance I'

$$I' = \frac{8}{2+6} = 1A$$

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P.D. across the cell.

$$V = 8 - 2\times1 = 6V$$

V = Potential Difference across balancing length.

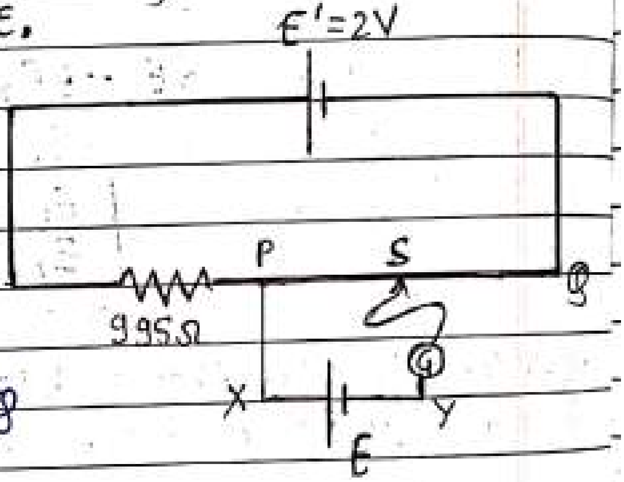
$$6 = I \times R_{AJ}$$

$$6 = 2 \times l \times x$$

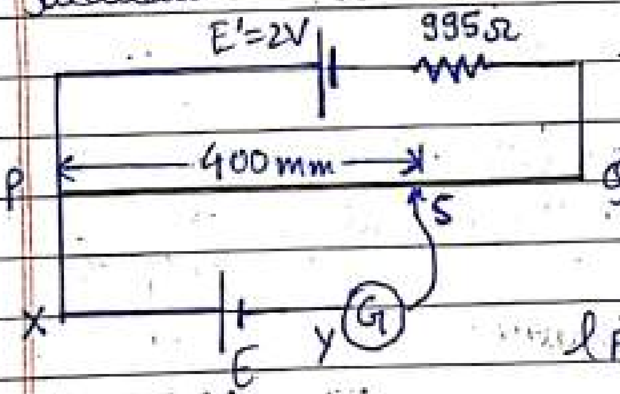
$$3 = l \times \frac{8}{100} \qquad \left[x = \frac{R_{AB}}{l_{AB}} = \frac{8}{100}\ \Omega/cm \right]$$

$$\therefore l = \frac{300}{8} = 37.5\ cm.$$

Q => The circuit diagram shows the use of a potentiometer to measure a small emf E, connected between the points X and Y. The cell of emf E' = 2V has negligible internal resistance. The potentiometer wire PQ is 1m long and has resistance 5Ω. The balance point S is found at 400 mm from P. Calculate the value of E.



Solⁿ => The given circuit can be redrawn as :->



$$l_{PQ} = 1m \qquad R_{PQ} = 5\Omega$$

$$I = \frac{E'}{R_{PQ} + R_{ext}} = \frac{2}{5+995} = \frac{2}{1000} = 0.002A$$

Resistance per unit length $x = \frac{5}{1} = 5\ \Omega/m$

$$l_{PS} = 400\ mm = 0.4\ m$$

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E = Potential Difference across balancing length

$E = I \times R_{PS}$

$E = I \times l_{ps} \times x$

$E = 0.002 \times 0.4 \times 5$

$E = 0.002 \times 2$

$\boxed{E = 0.004\ V}$

Q⇒ Heater 1 takes 3 min to boil a certain amount of water whereas heater 2 takes 6 min. Find the time taken if :-
(a) they are connected in series.
(b) they are connected in parallel.
Potential difference is V in all the cases.

Solⁿ ⇒ In series $\frac{1}{P_{tot}} = \frac{1}{P_1} + \frac{1}{P_2}$

In parallel $P_{tot} = P_1 + P_2$

as both heaters will generate same amount of heat to boil same amount of water.

$\therefore \quad H = P_1 t_1 = P_2 t_2$

$\Rightarrow \quad P_1 = \frac{H}{t_1}, \quad P_2 = \frac{H}{t_2}$

$P_1 = \frac{H}{3}, \quad P_2 = \frac{H}{6}$

(a) In series $P_{tot} = \frac{P_1 \cdot P_2}{P_1 + P_2} = \frac{\frac{H}{3} \times \frac{H}{6}}{\frac{H}{3} + \frac{H}{6}}$

$= \frac{H^2}{\cancel{18}_3} \times \frac{\cancel{6}}{3H}$

$P_{tot} = \frac{H}{9}$

as $H = P \times t \quad \therefore t = \frac{H}{P_{tot}} = \frac{H}{H/9} = 9$ min.

(b) In Parallel :

$$P_{tot} = P_1 + P_2 = \frac{H}{3} + \frac{H}{6} = \frac{H}{2}$$

as $H = Pt$ $\therefore t = \frac{H}{P} = \frac{H}{H/2}$

$$\Rightarrow \boxed{t = 2\,min}$$

Q ⇒ The emf of a storage battery is 90V before charging and 100V after charging. When the charging starts the current was 10A. What is the current at the end of the charging if the internal resistance (considered constant during the whole process) is 2 ohm.

Sol^n ⇒ Before charging $E_i = 90V$
After charging $E_f = 100V$
current at the end of charging $I_f = ?$

When charging starts, $I_i = 10A$

at the time of charging

$$V = E_i + I_i r$$

$$V = 90 + 10 \times 2 = 110V$$

Using the above eq^n again

$$V = E_f + I_f r$$

$$I_f = \frac{V - E_f}{r}$$

$$I_f = \frac{110 - 100}{2} = \frac{10}{2}$$

$$\boxed{I_f = 5A}$$